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AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

PROPOSED IMPROVEMENTS FOR LAND SURVEYS AND TITLE TRANSFERS

By Philip Kissam,[1] Assoc. M. Am. Soc. C. E.

Synopsis

The transfer of real estate in the United States is surrounded by difficulties out of all proportion to the values involved. This is particularly obvious when a comparison is made with the transfer of any other form of wealth. The unnecessary costs and delays, the investigations, surveys, and uncertainties of boundary location, the prolonged procedure used in proving title—all tend to reduce the negotiability of real property and to damage its value.

Surveying equipment, skill, and procedure have been developed which give results far beyond the accuracy required to eliminate boundary difficulties, and methods of recording title have already been successfully employed which eliminate delays in title search. It appears to the writer that the time has come for surveyors and title examiner to combine in a joint effort to find ways and means for employing these excellent tools already at hand.

The uncertainties of boundary location are caused basically by: (a) A lack of permanent, recognized, and correlated monuments from which surveys may originate; (b) increasing land values with no provision for more accurate surveys; (c) faulty descriptions (not erroneous); (d) blunders in description and surveys; and (e) the rules of adverse possession in force where accurate surveys and proper title records eliminate their usefulness.

The difficulties of title search are caused by: (a) The many and ever-increasing sources of information and records that must be studied; and (b) recurrent study of the same information and records.

To eliminate these difficulties the writer recommends:

(1) The establishment of State Systems of plane co-ordinates;

(2) A public officer to aid in, and permanently record, the determination of boundary lines for properties above an assessed unit value;

Note.—This paper was presented before the Surveying and Mapping Division, Jacksonville, Fla., April 21, 1938. Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **June 15, 1939.**

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(3) The assignment of certain members of the judiciary to a Land Court where title can be confirmed at the cost of only a small fee for legal service; and

(4) Simplification of title records by making a court decree final, with only a limited time for appeal, and by requiring all liens to be recorded with the evidence of title.

Introduction

One of the primary elements of civilization is the dedication of land areas to definite purposes. Devices for maintaining life, reducing labor, regulating men's relationships, providing enjoyment, or protecting spiritual values cannot be developed or used without the assignment of land. When the purpose of the dedication becomes clouded, or when the delineation of areas becomes vague, such confusion arises that progress is threatened. These truths are so fundamental that they are accepted without question. They are so well recognized that the first step upon the occupation of new territory is the assignment or division of land. They are so immutably fixed in men's minds that wholesale disregard of them is never contemplated.

Wherever the principle of private ownership obtains, possession of land carries with it the right to direct the use of the land under certain restrictions. It is therefore important, if progress is to be expected, that ownership and the limits of ownership shall be clearly defined.

It follows that land ownership must be a matter of public record, and that therefore it must be a function of government to maintain these public records. Not only must the records show who are the owners of the various land parcels and the conditions of ownership, but they must also clearly define the boundaries of the holdings. If land is to have its greatest value, it must be readily negotiable. Therefore, the public records must be quickly available and of unimpeachable accuracy. Each State must insure through legislative enactment that such conditions shall obtain, if confusion and serious economic loss are to be avoided.

How far has the United States progressed toward this goal? Unfortunately, in all the States, records of title and of liens affecting title are not only difficult of access but often require an expert to find them. They are frequently inconsistent with each other so that unless the title examiner's opinion is accepted, a court decree is necessary to bring them into agreement temporarily.

Are the conditions of the records of location any better? On the contrary, they are frequently worse. Recognition of position is usually based on hearsay, while size and shape may rest on the integrated opinion of many individuals. Under these conditions it can hardly be supposed that the parcels presumably owned will fit the existing ground, or that the location of a single parcel can be independently determined. Every land surveyor realizes this. Furthermore, the systems of recording are archaic, inadequate, and impractical. Obviously, therefore, the time has come for a change.

The Joint Committee

As civil engineers are continuously engaged in the design of structures and projects necessitating the acquirement and use of land, and as land surveying is also one of their functions, they should take the lead in initiating steps to bring about betterments. Furthermore, the locating of property is so intimately connected with its title that changes in procedure in regard to location must be accompanied by changes of procedure with regard to title. At the request of the Society, acting through its Surveying and Mapping Division, a Joint Committee has been formed with the American Bar Association, represented by the Real Property Division of its Section of Real Property, Probate and Trust Law. Presumably it will require some time for this Committee to formulate recommendations and make a final report.[2] Certain ideas brought out in Committee discussion are here described, although none of the statements in this paper represent action of the Committee, nor will they necessarily be included in the Committee's report.

Boundary Location

Whatever function a land surveyor is called upon to perform, he must first locate the boundaries of a parcel of land, even if a subdivision is required. If he can locate one corner and the direction of one line extending from that corner, a large portion of his difficulties are overcome. To aid him, the description of the parcel recorded in the deed probably is available. If this description is not independent, but is based on some other division of the land, he must resort to the earlier description. Sooner or later he must find mention of a point of beginning and some description of it. If this point is a natural or artificial monument, he is fortunate, provided check measurements made from other points he believes to be correct prove that the monument is in its original position.

Frequently the point of beginning is not marked, but is described as being so many feet along the center line or boundary of a public highway from some other public line. It is unusual if either of these lines can be found. Moreover, the surveyor never is sure that the present locations of these public lines, or his best judgment of where they should be located, will agree with the location assumed by the person who first described the point of beginning. In other words, more evidence must be brought to bear besides that found in the deed itself; and so many elements must be considered in each case that more judgment than skill is often required to establish even the initial point and direction.

To overcome this difficulty, recognized official survey positions must be provided by which property can be described and relocated. Systems of official monuments must be established and inter-connected by accurate surveys so that each monument is witnessed by all the others. When five or six monuments constitute the entire system, a balanced traverse loop will tie them together satisfactorily, and a plat showing the various adjusted traverse dis-

[2] "Land Surveys and Titles," First Progress Report of the Joint Committee of the Real Property Division, American Bar Association, and the Surveying and Mapping Division, American Society of Civil Engineers, *Proceedings*, Am. Soc. C. E., November, 1938, pp. 1879–1884.

tances and angles will serve as a record of their positions. Any municipality can safeguard the property in an important section of town by establishing a small system of this kind, recording the plat at the registry office, and maintaining the integrity of the monuments by re-surveys or check witness measurements at intervals. A purchaser of real estate in this area should be advised by his title examiner and his surveyor to demand a description in his deed based on these monuments.

It is better to substitute for the adjusted traverse data a rectangular co-ordinate system. The land surveyor need then only state the co-ordinates of each point mentioned in the description and name the co-ordinate system used. If possible, the co-ordinates of street intersections should be given on the plat. The title examiner will then have more information for tracing the title, and the public property will be better protected.

But when more than one co-ordinate system is established, it is inevitable that the systems will be extended until they meet, and then confusion will result. It is therefore desirable to have a few large systems rather than many small ones.

State Plane Co-ordinate Systems

The United States Coast and Geodetic Survey has established a system of plane rectangular co-ordinates for every State in the Union.[3] Each system is composed of one or more definite mathematical projections which are so well designed that they can be 150 miles wide and of any length desired without introducing material divergences, due to the earth's curvature, between actual measured distances or angles and those given by the co-ordinate positions. The usual maximum divergence is one part in 10 000, and for the greater part of each system the divergence is far less. The adaptation of these projections to the various States is chiefly the work of O. S. Adams of the Coast and Geodetic Survey, and the transverse Mercator projection was especially developed by him for this purpose.

Horizontal Control Surveys to Supplement the Fundamental Net

To use these systems it is necessary only to connect a survey with the Fundamental Triangulation Net of the United States. Wherever Coast and Geodetic Survey first- or second-order triangulation exists, this can be done immediately. The geographic positions of the triangulation stations can be reduced to plane co-ordinates by a comparatively simple computation. For many triangulation stations the Coast and Geodetic Survey has already made these computations. Of course it is better to amplify the triangulation net by horizontal control surveys supplemental to the Fundamental Net. This can be done by second-order traverse and, if plane co-ordinates are used, all reductions can be made by the usual plane surveying methods. Latitudes and departures are determined and the adjustment is made proportional to the lengths of the courses.

[3] See "Development of State Grid Systems," by O. S. Adams, *Civil Engineering*, January, 1937, pp. 33–37.

When control of this kind is developed, a sufficient number of control points presumably will be established in each town or locality so that surveys can be extended throughout the streets by local authorities.

Enabling Acts

Each State legislature should pass an enabling act, naming and describing the State system, so that State co-ordinates can be used in descriptions, and can be identified simply by naming the system. In New Jersey, Pennsylvania, and New York this has already been accomplished. Also, as soon as possible, in each State a survey bureau should be established to extend the control or supervise the establishment of control in various localities. Such bureaus should be the repositories of control survey data, should disseminate these data to the public, and should maintain the monuments. Pennsylvania has already established such a bureau by law; and in certain other States, control survey organizations act in this capacity.

With a State co-ordinate system, or other legally recorded co-ordinate system, the land surveyor has at his disposal a method of establishing a point of beginning and an initial direction which can always be precisely re-established. Such a system should not be confused with the Public Land System, for which the monuments (except in Alaska) are not inter-connected by precise surveys, and therefore are not susceptible of accurate replacement.

The method of describing property under the Public Land System has many advantages, particularly in tracing title. From the description alone previous title holders can be identified immediately, a procedure requiring infinite time and pains for rural districts in the East. The description gives the approximate location of the property, the description of the adjoining tracts, the approximate area of the parcel and a means of indexing the deeds by locality. However, it is impossible to define, in the description, the precise location of the property without resort to the marks originally established on the ground. They bear no mathematical relation to each other except in the roughest sense. When lost they can seldom be replaced in their original position and without the introduction of another survey it is impossible to determine rectangular co-ordinates for them. The State Plane Co-ordinate Systems provide all the advantages of the Public Land System and in addition provide a means of defining, precisely, the location of the land on the ground.

Descriptions Inconsistent with Each Other

There is a further difficulty, however. Should the surveyor try to mark a land parcel according to its description, it is very likely that he will encroach upon or fail to connect with abutting property. The shapes of the parcels described in the deeds seldom fit together to form the original tract from which they were separated. The blocks are not of the recorded length; the same line will have different lengths in adjacent lots; old boundary lines long established at inconsistent positions have become fixed by the rules of adverse possession, and the angles in possession are seldom as found in the deeds. Moreover, the adjacent properties are usually indifferently marked on the ground so that the

surveyor is completely in the dark if he attempts to disregard these inconsistent deeds.

There are five causes for these inconsistencies: (1) Lack of marked, recognized, and recorded positions, as already discussed; (2) variation in land values; (3) faulty descriptions (not erroneous); (4) blunders in descriptions and surveys; and (5) adverse possession (corollary of inconsistencies).

Variation in land value is a very natural reason for inconsistency. Survey accuracy is directly proportional to survey costs. Therefore, farm lands should not be surveyed with the same accuracy as business lots. Thus, precise measurements made in business areas usually disclose discrepancies in original surveys.

Although there is much diverse opinion relative to what constitutes a proper description, from the surveyor's point of view it should completely describe the size, shape, location, and orientation of the actual holdings. As such determination is next to impossible, many palliatives have been used, none of which has been satisfactory. Little reform can be expected until the causes of inconsistencies are removed.

Further, there exists a definite disagreement of purpose in descriptions. The title examiner needs a description which may be followed back accurately through previous transfers. The surveyor needs a description that can be used to mark accurately the present holding. The repetition of an inaccurate description copied from deed to deed is a boon to the title examiner, but is fraught with difficulties for the surveyor. A new and accurate description presents difficulties for the examiner. Mention of the abutters is good policy and aids both the examiner and surveyor, but without dimensions it is useful only to the examiner. It seems evident, therefore, that the solution of these difficulties must satisfy the needs of both examiner and surveyor.

How are these difficulties handled at present? If the surveyor is to make a proper location he must make a complete survey in the field of all the existing landmarks in the original tract or city block. He must abstract all the deeds for this area and, using his best judgment, work out a consistent solution which will satisfy the deeds, the physical location, and the requirements of geometry. If no objections are raised, his location usually stands, and, if court action ensues, he is well prepared for defense.

In such a procedure, however, the survey work is costly indeed. It cannot all be charged against the one parcel to be located. In substance the surveyor has worked out the solution for all the lots in the tract. Thus he has been obliged to invest a considerable fund in the area and he must protect his investment. He is forced to use secret marks so that he alone can profit by additional surveys in the same area, and he must maintain a complete office record of his solution and of the marks which show it on the ground. Actually, he has created a public benefit by his work, but this is available only when he is employed.

As long as surveys are made according to his solution and based on his system of marks, no difficulties will arise; but this is obviously impossible. The general public is unaware of the importance of his marks and records and will employ any surveyor offering a lower bid. The public streets and other

properties will not necessarily be made to agree with his solution and there is every likelihood that his office will not continue long enough to make his solution fully effective.

This leads to the second step in the reconstruction of land surveying methods. A public office must be maintained under the direction of an able land surveyor whose duty it is to see that when a deed is recorded the description is based on an accurate survey; that an adequate system of enduring monuments exists to mark the property permanently; and, finally, that the location is consistent with the surrounding holdings. This may appear to be an impossibility, but such is not the case. In Massachusetts, C. B. Humphrey, Land Court Engineer, performs all of these functions for land to be registered in the State.[4] After a land court decree, an official plat is drawn, so complete that even many years thereafter a surveyor entirely unacquainted with the area could mark the property.

The establishment of a system of this kind would be an immediate and important improvement. It would necessitate the appointment of a State surveyor-general with a deputy at each registry office. An unnecessary hardship would be created if it were made obligatory to bring all land under the system at once, as a certain fee would be required to cover the costs to the State. It would be better to require that, in order to record a deed to property assessed at perhaps $75 per front foot, there must be attached, in lieu of a description, a plat of the property issued by the deputy surveyor-general. This would stabilize property lines where the need was greatest, and require by law only an expenditure for surveys which should be made in any event under present conditions. Under this arrangement, the surveys would be made by private surveyors to whom the records of the deputy surveyor-general would be available. Their work would be checked, as is done in Massachusetts, and any control surveys necessary for solution where inconsistencies developed would be made by the surveyor-general's staff. Should suit be brought threatening any location prescribed, the surveyor-general would be required to defend the owner. As his records would probably be very complete, unless a blunder had been made, his evidence should be sufficient to bring a favorable decision.

Adverse Possession

With State co-ordinate systems, or even local co-ordinate systems, officially marked, recorded, and maintained by a proper bureau, the first difficulty would be eliminated, namely, indefinite initial position and direction. With an able surveyor-general there would be a gradual elimination of the inconsistencies due to: (1) Previous indefinite points of beginning; (2) old surveys made when the land had less value; (3) faulty descriptions; and (4) blunders in surveys and descriptions.

Adverse possession still remains, however, as a factor with disruptive possibilities. It is based on a principle of major importance. Possession is said to be nine-tenths of the law; and it is very proper that it should be so consid-

[4] "State-Wide Surveying Practice in Massachusetts: A Symposium," by Elmer C. Houdlette and Clarence B. Humphrey, *Proceedings*, Am. Soc. C. E., November, 1938, pp. 1847–1878.

ered, as its effect is to stabilize ownership. It protects the owner against the innumerable possibilities of loss of title to all or part of his land which may arise under the present system; but it is an admission of the failure of the system. If a proper system is established, adverse possession becomes a threat rather than a stabilizing influence. It can be eliminated only when a system is established which is so nearly perfect that there can be no fear of unknown claims arising. Such a system must require very careful and complete study of conditions affecting the title, together with the citation and hearing of all persons interested. A court decree would also be required, which, when issued, would constitute a barrier to any inquiry into title prior to that time; but the effect of the decree would not be lasting and a new or supplemental decree would be required whenever the property changed hands. To overcome this defect and reduce the difficulties connected with title search, the Torrens System was devised, and in some form has been adopted in a number of States.

The Torrens System

Briefly, the Torrens System provides the means of obtaining a certificate of ownership when a petition is presented in court. The original certificate is kept at the office of registry, and a duplicate is given to the owner. This certificate is complete evidence of title. On it must appear a record of all liens against the property, except Federal liens and recent tax liens. References are given to the proper documents. Liens not appearing are invalid. When the property changes hands, a new certificate is issued upon presentation of the owner's certificate and the filing of the deed. In order to procure the first certificate, the owner must apply to the court. The court appoints an examiner who makes a thorough examination. Upon receiving the examiner's report, all persons possibly interested are cited, and after a short interval the court holds a hearing and issues a decree. The recorder immediately issues the certificate according to the decree.

The weakness of the system is twofold. Usually no attempt is made to find the location of the property, nor to register its location. Also the petitioner bears the cost of registration, which is of no more advantage at the time than an ordinary court decree. Although forms of the Torrens System have been adopted in many States, the extent of their use is negligible in spite of the fact that all of the serious difficulties of search are eliminated, as is also the fear of unknown claims arising. Most of the Torrens Acts also establish an assurance fund to protect the petitioner if loss is incurred. It is questionable whether this is a proper governmental function or should not be left to private title companies.

The Title Companies

A properly operated title company offers considerably more security than the Torrens System. It brings about, unofficially, the elimination of location inconsistencies and arduous title search. Thus the owner is usually protected against the threat of subsequent redetermination of his boundaries as well as against any unknown claims arising. Such companies should maintain com-

plete survey records of all solutions, as well as complete abstracts of title for the tracts of land where they operate, although maintenance of the survey records is sometimes left to a practising surveyor. With these records, location and title search are quickly and accurately performed.

There is no assurance, however, that any company will be the sole repository of all the survey records for any tract. Inconsistencies may arise between the solutions of two companies. Also, the title company system provides no means of clearing a clouded title, and frequently where location solutions, or title difficulties, require a court decree, the costs of both the insurance and the decree prohibit clearing the title.

In Massachusetts a splendid system has been in operation since 1898, with a special court, called the Land Court.[4] Under this plan are included all the elements necessary to the successful administration of land ownership.

Conclusion

It appears that certain elements are necessary to improve land surveys and title transfers. These elements can be obtained gradually by progressive steps, or they can be created simultaneously:

(*a*) Of first importance are the State Systems of Plane Co-ordinates. They provide precise initial positions and directions so as to give descriptions definite meaning and property definite position. These systems require enabling laws, and the establishment of supplemental control surveys and a State survey bureau to administer them.

(*b*) Inconsistencies between deed descriptions may be eliminated by the appointment of a surveyor-general to supervise the surveys and descriptions for properties above a certain unit value, when the deeds are to be recorded. He must make solutions for these inconsistencies in each tract required, maintain public records of his solutions, and defend his solutions in court.

(*c*) A land court should be established; after proper citation of interested parties, and a review of examination of title, it will issue a decree which will include a description approved by the surveyor-general, will constitute a barrier to any prior inquiry into title, and will be filed at the office of the register of deeds for public reference.

(*d*) Finally, there is the question of simplification of current records. Whether this can best be handled by title registration, or by title companies, appears to be a question. Certainly a title company can render more varied and comprehensive service. Probably the answer lies in the ability, strength, and administration of the title companies themselves.

None of these essential elements is revolutionary or especially costly. State co-ordinates provide a basis for the accurate maps and plans so necessary for engineering progress, afford a method of compiling map data for planning in detail and over large areas, and justify their cost even when used only for control for tax maps.

A surveyor-general would perform a service which is required at present. He would function only by bringing under one head all the unco-ordinated

efforts of many land surveyors privately attempting to make solutions for boundary tangles, which, to be effective, must be of public record.

The land court is only an assignment of certain of the present judiciary to a court which will handle all real estate title matters. It has been estimated that a decree to confirm title would cost the State little, or nothing, and the petitioner about $25 for legal fees.

Title companies under such a system could insure with less risk and therefore for lower fees. The difficulty of maintaining their records would be reduced to a minimum.

With these objectives as a goal and with the concentrated effort of all who understand these problems and realize their importance, the attainment of proper methods for land surveys and title transfers cannot be far in the future and the people can look forward with considerable hope that the present inadequate, archaic, costly, and wasteful methods of determining land ownership have not long to last.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

HYDROLOGY OF THE GREAT LAKES

A SYMPOSIUM

Note.—Written comments are invited for immediate publication; to insure publication, the last discussion should be submitted by **June 15, 1939.**

TYPICAL QUANTITATIVE ANALYSIS AS APPLIED TO LAKE SUPERIOR

By C. R. Pettis,[1] M. Am. Soc. C. E.

Synopsis

A method of making a quantitative analysis of the hydrology of the Great Lakes, as illustrated by a detailed study of the hydrology of Lake Superior, is presented in this paper. Certain deductions are made, based on evidence from available data. By means of these deductions, values are obtained for the evaporation from the lake surface. Knowing the evaporation, the underground flow to the lake and the land losses can be computed.

The validity of the method has been checked by evaporation experiments which are described in the companion paper of this Symposium.

The conclusion reached is that the evaporation and the underground flow are greater, and the land losses are less, than the values that have been generally accepted.

Introduction

The hydrology of the Great Lakes has been the subject of several studies: Thomas Russell, of the United States Lake Survey (1906); the late John R. Freeman, Past-President and Hon. M. Am. Soc. C. E. (1926); Robert E. Horton, M. Am. Soc. C. E., and the late Carl E. Grunsky, Past-President, Am. Soc. C. E. (1927); and John F. Hayford and J. A. Folse (1930). Most of them assumed a value for evaporation determined from ordinary land pans; Mr. Freeman assumed that the underground flow to the lake was zero; all of their results differed considerably from those obtained by the present analysis.

A quantitative analysis of the hydrology of Lake Superior must account for the disposal of all the water that enters the main body of the lake, for such periods of time as may be considered. In any given period of time, the total quantity of water that enters the lake, as precipitation on the surface of the lake, river flow and underground flow, will be equal to the evaporation from the lake surface plus the flow through St. Marys River, plus the change in storage, as indicated by lake levels in the same period of time. If the river flow to the lake is known, and the underground flow to the lake can be determined, the tributary land drainage basin can be included in the analysis by using the relationship: Over a period of several years, the rainfall on the land basin is equal to the river flow to the lake, plus the underground flow to the lake, plus the land losses. The term "land losses" includes all land precipitation that does not reach the lake; presumably the land losses consist primarily of land evaporation and transpiration, or other vegetative losses.

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Analysis

Formulas.—The following formulas apply to any given period of time:

$$P + R + U = O + S + E \ldots\ldots\ldots\ldots (1)$$

and

$$B = R + U + L \ldots\ldots\ldots\ldots (2)$$

All of the terms in the equations represent volumes of water in the period of time under consideration. They are defined as follows:

P = the rainfall on the surface of the lake, and becomes an immediate addition to the body of water in the lake as soon as it falls. It is assumed that the depth in inches of rainfall on the lake is the same as the depth of rainfall on the land; it was decided that this assumption would give better results in this analysis, because the evidence of a slight difference between P and B is conflicting.

B = the rainfall on the tributary land area. This value was computed from records of 59 stations of the United States Weather Bureau in the area, by taking a weighted average for the sections into which the area was divided.

R = the run-off into the lake from the surface streams as determined from stream-flow records of the United States Geological Survey and other sources, using a method of weighted averages, based on a personal reconnaissance of the area, 49% of which is covered by stream-flow records. The value of R, as a whole, is considered fairly accurate.

U = the net underground flow into the lake, which is not known, and is to be determined.

O = the outflow from the St. Marys River, which is based on careful records kept by the U. S. Engineer Department.

S = the change in storage in the lake, as indicated by gage records, being plus for a rising lake and minus for a falling lake. The gage records are kept by the United States Lake Survey.

E = the net evaporation from the surface of the lake, and is considered in the usual sense of evaporation minus condensation.

L = the difference between the rainfall B and the surface and underground flow into the lake ($R + U$), for the period under consideration, as given by a transposition of Equation (2).

The period of time used in most of this analysis is one calendar month. Symbols P, R, U, O, S, and E all denote quantities of water that enter or leave the main body of the lake during the month in question. Symbol B is the precipitation on the land area for the same month. In winter much of the precipitation remains on the surface of the ground and does not enter the lake, either as surface flow or underground flow, until the spring thaw. Throughout the year, the land precipitation, B, is subject to a delay before entering the lake, due to both surface and underground storage effects. For this reason, the value of L for any particular month, as obtained by Equation (2), will not represent the land losses (evaporation and transpiration) which pertain to the precipitation of that particular month, the land losses being that portion of the precipitation that does not ultimately enter the lake. Over a number of years,

the values of L for the period will represent the land rainfall that does not reach the lake; that is, the land losses, primarily land evaporation and transpiration.

There are two equations and three unknowns, E, U, and L. A rigid solution is impossible; but by making certain assumptions in accord with the present knowledge of the subject (and which, in most cases, seem to be the only reasonable assumptions possible, in view of the evidence) it is possible to arrive at an approximate solution. If one unknown is determined approximately, a rigid solution is possible for the other two unknowns, although the results necessarily cannot be more accurate than the approximate determination of the first unknown.

As far as is known, none of the lakes freezes over entirely in the winter. "Open-water" evaporation refers to the rate of evaporation from the part of the lake that is not covered by ice. The actual evaporation from a lake in the winter will be less than the "open-water" evaporation by an amount that depends on the extent of ice that is formed.

Since January, 1937, evaporation stations have been maintained by the Lake Survey at Detroit, Mich., Kewaunee, Wis., Duluth, Minn., and Buffalo, N. Y. An effort is made to keep the water temperature the same as the temperature of the water in the open lake, which makes them different from ordinary land-pan experiments.

Results reported by Mr. Hickman in this Symposium are based fundamentally on the evaporation experiments. In this paper, a mathematical analysis is made of data which is entirely different from that used by Mr. Hickman, and monthly values of the evaporation on Lake Superior are obtained. The results of the two independent solutions check rather closely; the results obtained by investigators who neglected underground flow do not check with the evaporation experiments. When the evaporation has been determined, by any method, the values of U and L can be determined mathematically from Equations (1) and (2). The analysis and solution given in this paper were made before the evaporation experiments were begun. It is evident from Fig. 1 that the remainder of this paper, with slight modifications, might be considered to be a solution and discussion based on the evaporation experiments.

From the data and Equations (1) and (2), monthly values of $(E - U)$, $(E + L)$, and $(U + L)$ can be obtained which will be as accurate as the data upon which they are based.

As a matter of convenience, in all computations, the water that will cover the land area tributary to Lake Superior to a depth of 1 in., will be used as a unit of volume. Unless otherwise indicated, all data will be given in the same unit. The land area of Lake Superior is 49 078 sq miles. One unit, or 1.0 in. of water on this area, will represent a certain definite quantity, which is practically 114 billion cu ft. The water area of Lake Superior is 31 817 sq miles. The ratio of land area to water area is 1.54. If a 1-in. depth of water on the land were placed in the lake, the lake level would be raised 1.54 in. Inches of depth on the land can be converted to inches of depth on the lake by multiplying by 1.54; and, conversely, inches of depth on the lake can be converted to inches of depth on the land by dividing by 1.54.

In this analysis, the following assumptions are made:

(*a*) It is assumed that the net underground flow into the lake, U, is always positive; that is, there will be no monthly period in which there is a net flow of underground water away from the lake. This assumption is definitely indi-

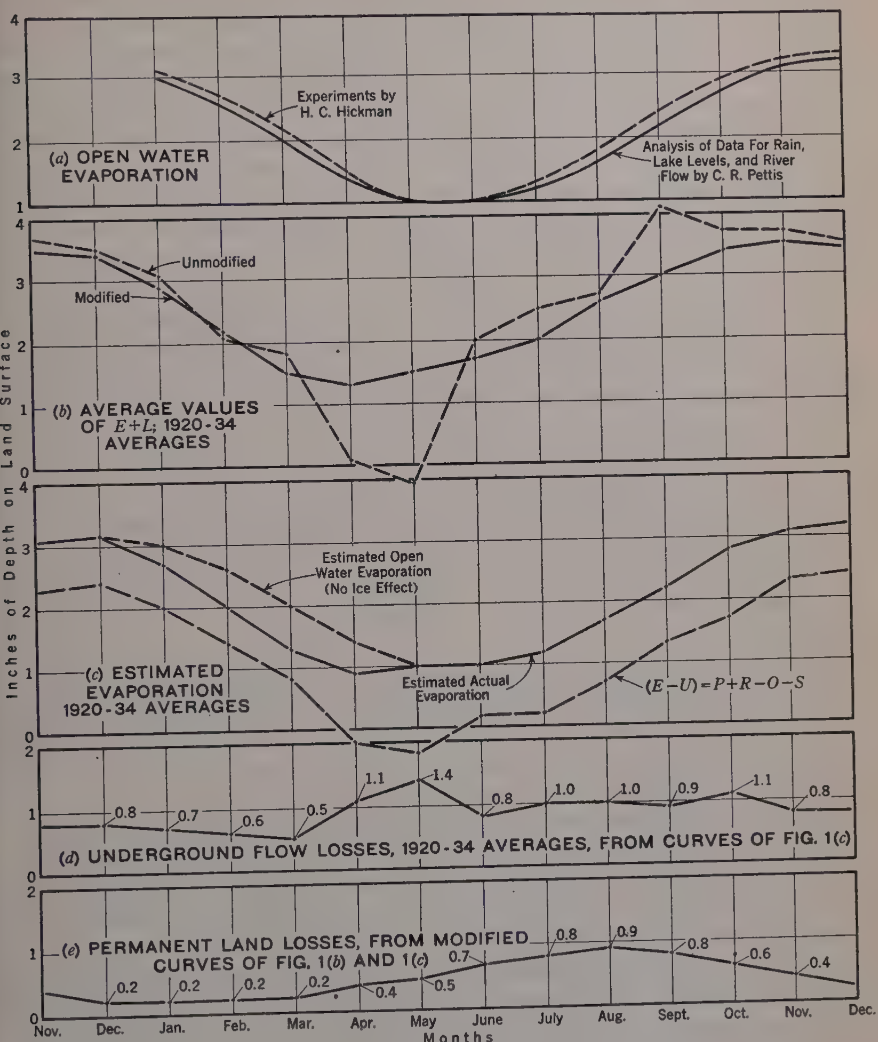

FIG. 1.—HYDROLOGICAL DATA APPLYING TO LAKE SUPERIOR

rated by the analysis of curves in this study. The flow of underground water even in the winter has been indicated by Mr. Horton who states:[2] "In case of sandy soils with low field moisture capacity, infiltration takes place at a relatively high rate even when the ground is frozen." Messrs. H. K. Burton

[2] National Research Council Report, 1936, p. 351.

and J. Cecil Alter state:[3] "The mountain snow-layer melts continually from the bottom, the moisture leaching steadily into the soil."

(*b*) It is assumed that the normal open-water evaporation curve for monthly periods will be approximately a sine curve with a periodicity of one year. The curves for air and water temperatures, relative humidity and wind, which are the factors that are supposed to affect evaporation, are approximate sine curves with periodicities of one year. There is some variation from normal in the evaporation for particular months; but, in general, the variation in evaporation from the normal for a particular month is not large. According to H. K. Barrows,[4] M. Am. Soc. C. E., for any given month there is, in general, a greater range in the rate of precipitation than in the rate of evaporation. For that reason less time is required to obtain data of average monthly evaporation sufficiently accurate for use in storage estimates. Other engineers have noted the same condition.

(*c*) The $(E + L)$-curve in Fig. 1(*b*), with obvious modifications, is approximately a sine curve. The curve representing the permanent land losses, when distributed over the months in which the losses occur, should be something like a sine curve; the numerical variations in the permanent (distributed) L-curve are probably less than the variations in any of the other curves.

If the $(E - U)$-curve is plotted by months (see Fig. 2, with Tables 1 and 2), and compared with the rainfall B-curve, it is apparent at once that, in practically every case when the weather is not freezing, a high value of rainfall in one month will be followed by a low value $(E - U)$ in the following month, or sometimes in the second month. This is true for each of the Great Lakes. This relation between B and $(E - U)$ must be accounted for by one of two hypotheses:

(1) The high rainfall in one month causes a radical change in the evaporation for the following month. This assumption is directly contrary to present theories on the subject of evaporation; it is inconsistent with the previous assumption in regard to the regularity of evaporation by monthly periods; and, therefore, this assumption is rejected as being untrue.

(2) High rainfall in one month will be followed by an increase in underground flow which will be most noticeable in the lake in the following month or, say, five weeks later. There is nothing in this assumption contrary to the present knowledge of the subject; it seems reasonable and, therefore, it is adopted for the purpose of this analysis. It may be noted that a high rainfall in the summer will have a greater effect on the underground flow in the next month than the same rainfall in the late fall or winter, which is consistent with the present engineering opinion that underground water flows more freely when the temperatures are warm than when the temperatures are cold (see Table 1).

There is a considerable flow, R, from the surface streams through the winter; November, 0.9 in.; December, 0.7 in.; January, 0.6 in.; and February, 0.5 in. Since the ground in the Superior basin is ordinarily covered with snow during these months, it is assumed that most of this winter flow comes from under-

[3] National Research Council Report, 1936, p. 534.

[4] "Water Power Engineering," by H. K. Barrows, p. 83.

ground sources. The surface streams can tap only the portion of the underground water that is above certain levels; the underground water below these levels can reach the lake only through underground channels.

The winter underground flow is deduced by the following analysis: The 6-yr period 1920–1925 was a period of relatively small annual rainfall (average 26.6

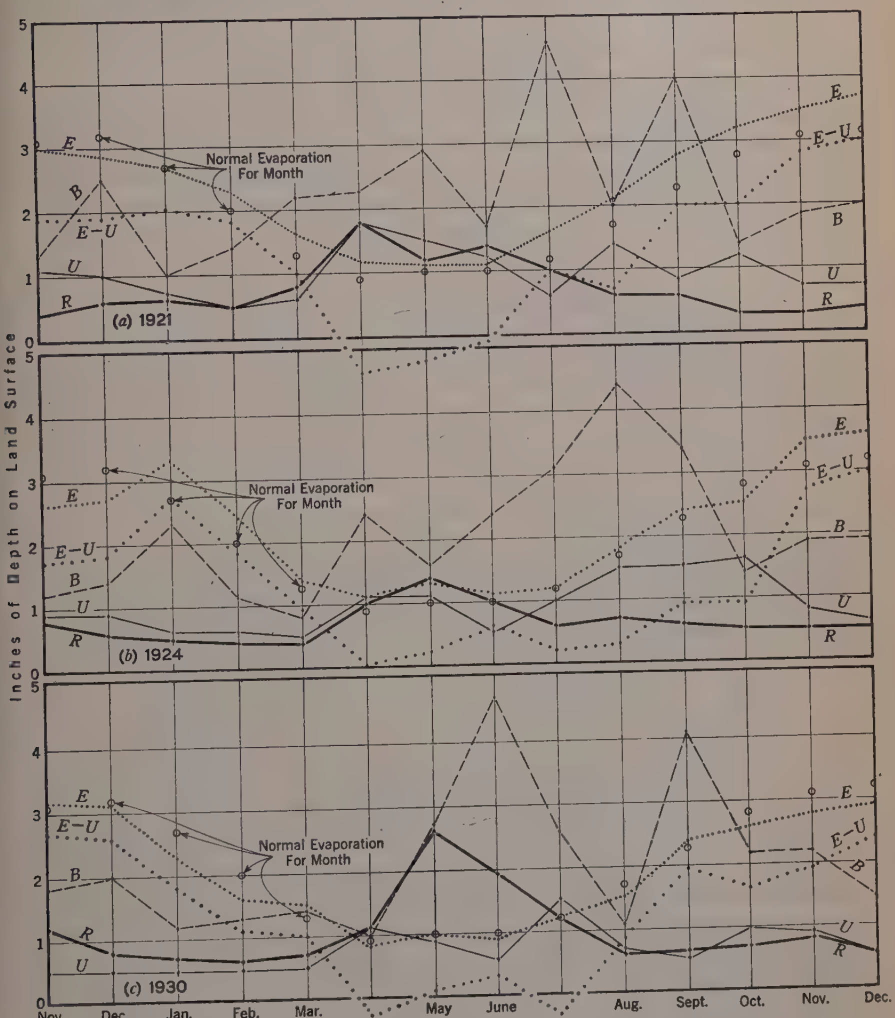

FIG. 2.—ESTIMATED EVAPORATION AND UNDERGROUND FLOW; LAKE SUPERIOR

in.); and, the 6-yr period 1926–1931 had a greater annual rainfall (average 29.8 in.). The average annual temperature for the two periods varied by less than 0.5° F. If the winter evaporation, December to March, for the two periods is assumed to average the same, the values of ($E - U$) indicate that the underground flow from December to March for the second period was greater than that for the first period by 0.9 in.

TABLE 1.—TEMPERATURE AND WIND RECORDS; LAKE SUPERIOR; CORRESPONDING TO FIG. 2.

Description	November	December	January	February	March	April	May	June	July	August	September	October	November	December
(*a*) 1921; A WARM YEAR; WINTER WARM, WITH LITTLE ICE														
Average 15-year normal air temperature, in degrees Fahrenheit	31	18	13	15	24	37	50	60	65	63	57	45	31	18
Actual monthly temperature, 1921, in degrees Fahrenheit...	31	23	20	20	26	42	51	64	72	64	60	47	27	19
Wind, percentage of normal over a 15-year period...........	110	120	100	100	120	140	100	100	100	100	120	100	82	110
Evaporation (+ = above normal)....................	...	...	...	...	...	...	...	...	...	...	...	...	...	...
(*b*) 1924														
Average 15-year normal air temperature, in degrees Fahrenheit	31	18	13	15	24	37	50	60	65	63	57	45	31	18
Actual monthly temperature, 1924, in degrees Fahrenheit...	36	28	5	17	26	36	45	57	62	62	53	51	31	10
Wind, percentage of normal over a 15-year period...........	90	100	110	100	90	120	120	100	100	112	100	90	110	120
Evaporation (+ = above normal)....................	−	−	+	...	...	+	+	...	...	+	+	−	+	+
(*c*) 1930														
Average 15-year normal air temperature, in degrees Fahrenheit	31	18	13	15	24	37	50	60	65	63	57	45	31	18
Actual monthly temperature, 1930, in degrees Fahrenheit...	28	17	8	18	24	39	51	62	65	67	56	44	34	21
Wind, percentage of normal over a 15-year period...........	90	90	90	90	110	90	100	90	100	90	110	90	100	80
Evaporation (+ = above normal)....................	...	...	...	−	+	−	...	−	...	−	+	−	−	−

TABLE 2.—HYDROLOGICAL DATA; LAKE SUPERIOR, CORRESPONDING TO FIG. 2.

Description	1921	1924	1930
Net Evaporation, E, from the Surface of the Lake:			
November to October, inclusive....................	25.6	23.8	22.1
Run-off, R, into the Lake from Surface Streams:			
January of following year....................	0.5	0.5	0.3
February of following year....................	0.5	0.5	0.3
March to February, inclusive....................	9.4	8.2	11.3
Net Underground Flow, U, into the Lake:			
January of following year....................	0.7	0.6	0.5
February of following year....................	0.6	0.6	0.5
March to February, inclusive....................	11.8	11.4	9.2
Rainfall, B, on Tributary Land Surface:			
November to October, inclusive....................	27.3	25.5	26.1
$R + U$: March to February, inclusive....................	21.2	19.6	20.5
$L = B - (R + U)$: From two items above....................	6.1	5.9	5.6

The rainfall in August, September, and October is a major (but not the sole) factor affecting winter underground flow. From Equation (2), the rainfall available for underground flow is $(B - R - L)$. Taking an average value of L for the given months from Fig. 1(*d*), and using known data, $(B - R - L)$ August to October, for the first period, was 3.4 in., and for the second period, 4.8 in.; the average value was 4.1 in. for both periods combined; and, the difference between the low period and high period was 1.4 in. Assuming that the difference in available rainfall (1.4 in.) is to the difference in winter underground flow (0.9 in.) as the average available rainfall (4.1 in.) is to the average underground flow, then the average underground flow, December to March, is 2.6 in. This was distributed to months as follows: December, 0.8; January, 0.7; February, 0.6; and March, 0.5.

The average monthly values of $(E + L)$ for the 15-yr period, as computed by the formula:

$$E + L = P + B - O - S \qquad (3)$$

are shown by the curve in Fig. 1(*b*). For any period long enough to equalize the underground-storage and lake-storage effects, $(E + L)$ will represent the quantity of water that falls on the total area and which has not passed out of the area through St. Marys River.

In the $(E + L)$-curve shown in Fig. 1(*b*), if more water than usual goes into underground (or snow) storage in any particular month, then L for that month —and also $(E + L)$—will contain not only the permanent land loss for that month, but also a temporary loss, which represents water that has gone into storage, and which will decrease the formula values of $(E + \mathrm{L})$ for the later months in which it comes out of storage. It is evident that certain adjustments may be made in the $(E + L)$-curve to obtain a modified $(E + L)$-curve which will represent the values of $(E + L)$ with land losses distributed to the months in which they actually occur. It will be noted that there is a considerable sag in the unmodified $(E + L)$-curve in April and May (see Fig. 1(*b*)). This sag is due to the fact that, in these months, considerable water that had fallen during the winter months, in the form of snow, becomes available for surface flow and underground flow, causing a negative value of L.

The years 1924 and 1931 had the smallest winter precipitation of any years in the 15-yr period under study. The April value of $(E + L)$ for 1924 was 1.1, and the May value of $(E + L)$ for 1931 was 1.4. These values are only slightly less than those adopted for the modified $(E + L)$-curve for these two months. In comparing the $(E + L)$-curve and the modified $(E + L)$-curve, it will be noted that 3.0 in. is gained in the spring thaw. This represents the water that has gone into storage in the preceding months, and which comes out of storage at the time of the thaw. The difference between the $(E + L)$-curve and the modified $(E + L)$-curve (Fig. 1(*b*)) is caused by the varying effects of storage on the values of L.

If the modified $(E + L)$-curve is compared with a sine curve, it will be noted that there is a slight sag in the winter months, ending about May. This is caused by a certain area of ice forming near the shores of the lake in the winter, which reduces the evaporation. Lake Superior very seldom, if ever, freezes

over entirely. It is doubtful if as much as half of the lake has frozen over during the period 1920–1934.

Before an attempt is made to analyze the curves for individual years, it is desirable to ascertain the approximate shape of the E-curve for an average year. The average monthly values of $(E - U)$ for the 15-yr period are plotted in Fig. 1(c). Since it is assumed that U is positive at all times, all points of the E-curve must lie above the corresponding points of the $(E - U)$-curve; in each case it is by an amount that is equal to the underground flow for the particular month. Previously, herein, average values of U were assumed for December to March. This assumption yields points on the E-curve for December to March, with the maximum point of the E-curve in December. For open-water evaporation, the minimum point of the curve can be assumed to occur about six months from the maximum point, or about June. The evaporation experiments, reported in this Symposium by Mr. Hickman, indicate that the minimum monthly evaporation to be expected on Lake Superior will not be less than 1 in. (land area unit). It will be assumed that the minimum evaporation in May and June is 1.0 in. per month. The open-water evaporation curve can now be obtained by drawing an approximate sine curve between the high points and the low points, and through the other points determined. The actual evaporation curve, or the full-line curve in Fig. 1(c), is obtained for the month of April by considering that a reduction in evaporation, due to ice, continues through this month.

From the E-curve (Fig. 1(c)), the $(E - U)$-curve (Fig. 1(c)), and the modified $(E + L)$-curve (Fig. 1(b)), the average monthly values for U and the permanent distributed values of L can be obtained. The curves for U and L are shown in Fig. 1(d).

The U-curve is higher in the warm months, and lower in the winter. The highest point is in the spring thaw, when the melting snow is added to the rainfall. The curve reflects the higher rainfalls in July and September, by increased flow in August and October.

The L-curve (Fig. 1(d)) may be considered to be caused primarily by land evaporation and transpiration. The shape of the L-curve suggests that the land-evaporation curve is somewhat similar to that of the ordinary pan-evaporation experiment, being higher in the summer, and, that transpiration losses vary with the activity of vegetable growth. Attention is called to the fact that "land evaporation," as the term is used in this paper, means the evaporation proper (water vapor leaving the land surface), minus condensation (water vapor absorbed from air by soil). According to some investigators, the rate of condensation is considerable.

An estimate has been made of the monthly evaporation for each one of the fifteen years in the period 1920–1934; the result of this detailed analysis for three typical years is shown in Fig. 2, with Tables 1 and 2. The methods used in the analysis have been evolved as the result of an extended study.

The average air temperature and relative wind velocities were computed for each month from data supplied by the U. S. Weather Bureau (see Table 1); a decided departure of either one of these factors from normal for any month will generally have a noticeable effect on the evaporation for that month.

Relative humidity was computed for each month; an effort was made to relate the monthly values of relative humidity to the corresponding data for evaporation, but the conclusion was reached that the monthly relative humidity is more the result of the evaporation than it is the cause of evaporation. For this reason, relative humidity was not used in the analysis; which is equivalent to making the assumption that, for the 15-yr period, the values of relative humidity remain normal according to the period of the year considered.

In the analysis, the first step is to plot monthly values of the $(E - U)$-curve; then values of E are assumed for each month, which best comply with the following conditions:

(1) If air temperature is normal for the month, and has not been abnormal for the months immediately preceding, and if the wind is normal for the month, E should be nearly normal for the month, as shown by the E-curve in Fig. 1(*c*).

(2) A sudden lowering of the temperature for one month will tend to cause a higher value of E for this month; this accords with Dalton's law, since water temperature tends to follow the air temperature, but with a considerable lag.

(3) If air temperatures are above normal for a few months (say, three or more), water temperature will probably be above normal, and E will be increased.

(4) In a cold winter, E will tend to decrease, due primarily to a larger area of ice formed on the lake.

(5) A relatively high wind will cause a higher value of E.

(6) The normal winter values of U previously deduced are assumed to apply, except where modification may be indicated by some of the other factors.

(7) The values of E in the spring months should be such as to reflect the spring thaw in the values of U; and, for different years, the values of U in the spring thaw should have some relation to the winter precipitation.

(8) There should be a relation between the precipitation and the values of U. Abnormally high or low values of B, except in the winter, should be followed in general by high or low values of U within a month or two.

(9) In some cases, the conditions will be conflicting; a few minor discrepancies are to be expected, due to slight errors in the data, or to the method of treating relative humidity.

The application of the foregoing principles to three typical years is shown in Fig. 2, with Tables 1 and 2. The climatic data for each month are plotted on the vertical line beneath the given month. In Table 1, the first line of each sub-table is the average (15-yr) air temperature for the corresponding month. The second line gives the average air temperatures for the months in question. The effect of the air temperature is judged by a comparison of the first two lines. The third line gives the wind velocity for each month as compared with the average (15-yr) wind velocity for that month. The fourth line shows by $(E +)$ or $(E -)$ certain months in which the temperature and wind indicate that the evaporation for the particular months will be appreciably above or below normal for those months.

The $(E - U)$-curve is computed by a transposition of Equation (1). The R-curve is drawn from the tabulated values obtained from the Geological

Survey data. The small circles in the figures indicate the normal evaporation for respective months, as previously determined. The E-curve indicates the estimated monthly evaporation, giving consideration to climatic factors, and to the conditions previously enumerated. The value of U for each month, as shown by the U-curve, is determined by taking the distance between the E-curve and the $(E - U)$-curve.

For convenience in making comparisons, the B-curve, for rainfall on the land, is plotted on the same sheet.

Values of E, R, U, $(R + U)$, B, and L, are presented in Table 2, L being computed from a transposition of Equation (2), using different periods for the several factors, in order to reduce the error in land losses, L, due to temporary storage effect. The twelve-month period, November to October, inclusive, was used for B in this formula; and, March to February was used for R and U, for the following reasons: The precipitation (in the form of snow), from November to February, has a much greater effect on R and U in the spring thaw after March than it does on the winter flow of R and U. The winter flow is affected principally by the rainfall in October and the preceding months, so that the rainfall during the period used in the formula for B reaches the lake principally in the period used in the formula for R and U; and the values of L computed by this method are fairly uniform for most years, since the storage effect will be small unless conditions are somewhat abnormal. The correlation between climatic factors and evaporation, and between rainfall and underground flow can be observed in Fig. 2.

The year 1921 was an unusually warm year, and the evaporation was high (25.6 in.). The high evaporation for this year is further substantiated by a correspondingly high value of $(E + L)$, which can be computed from known factors by Equation (3). The winter of 1930 apparently had more ice than either 1921 or 1924. It was the only year of the three with a November-to-March temperature below normal. The year 1921 had the highest winter precipitation, from November to March, and it also had the largest underground flow in the spring thaw in April and May. The curves in Fig. 2 indicate that there is a fairly close correlation between rainfall, B, and the total flow to the lake, $(R + U)$. The distribution of B between R and U, evidently, depends on other factors than those that have been shown.

Next, consider the possibility of errors in the data, and the effect that such errors might have on the values of E, L, and U. For the purposes of this analysis, it is believed that the values of O and S may be accepted as correct, and that minor errors are more probable in the values of P, B, and R. The following statements are believed to represent the principal effects of possible errors:

From Equation (1), $(R + U) = O + S + E - P$. If the values on the right are assumed to be correct, an error in R will cause an equal error in the opposite direction in U; and to a considerable extent, errors in R will be compensated by opposite errors in U, without affecting the values of E and L.

From Equation (1), $E = P + (R + U) - O - S$. If O and S are assumed correct, and errors in R are compensated in the determined values of U, an error in P will cause an equal error in the same direction in E.

From Equation (2), $L = B - (R + U)$. If errors in R are compensated in the determined values of U, an error in B will cause an equal error in the same direction in L.

The collection of water in rain gages is subject to wind effects, and the water in most gages is subject to some slight evaporation. It is believed that both B and P may be slightly greater than the Weather Bureau data used, and that the values of E and L may be slightly larger than the determined amounts.

Summary and Acknowledgment

The results of this analysis for Lake Superior are as follows:

Land area		Inches of depth on land area
Normal annual rainfall on land is		28.2
of which land losses are		5.9
The remainder		22.3
Reaches the lake as:		
Surface run-off	11.6	
Underground flow	10.7	

Lake area		Inches of depth on lake area
Normal annual rainfall on lake is		28.2
Annual surface run-off to lake is		17.9
Annual underground flow to lake is		16.5
Total addition to lake in one year is		62.6
Of which annual evaporation is	35.6	
And outflow from St. Marys River is	27.0	

The writer has received assistance from a number of sources, especially from Messrs. Sherman Moore, H. C. Hickman, Jun. Am. Soc. C. E., L. D. Kirshner, and F. W. Townsend, of the Lake Survey, and Professors C. O. Wisler and W. C. Hoad, Members, Am. Soc. C. E., and E. F. Brater, Jun. Am. Soc. C. E., of the University of Michigan, at Ann Arbor, Mich. The data form a part of a thesis[5] by the writer, entitled "Hydrology of the Great Lakes," presented to the University of Michigan in 1938, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

[5] A copy of the thesis, including all observations and other data, has been placed on file in the Engineering Societies' Library, 33 West 39th Street, New York, N. Y.

EVAPORATION EXPERIMENTS

By Harold C. Hickman,[6] Jun. Am. Soc. C. E.

Synopsis

The evaporation experiments reported in this paper were conducted at Duluth, Kewaunee, Detroit, and Buffalo, for the purpose of providing a basis for estimating the evaporation from the water surfaces of the Great Lakes. In order to be applicable to local conditions, an effort was made to keep the temperature of the water in the pans approximately the same as the temperature of the water in the open lakes. Data were secured which cover a greater variety of combinations of air and water temperatures and wind velocities at the evaporation pans, than any known heretofore. In applying these data to the Great Lakes it was assumed that the evaporation from the open lakes, for a given air temperature, water temperature, and wind velocity, would be the same as the observed evaporation from the experimental pans for the same combination of air temperature, water temperature, and wind velocity.

The readings were analyzed and plotted on two charts, from which an estimate of the monthly (or daily) evaporation from the Great Lakes can be made. In these charts it has been assumed that the relative humidity and barometric pressure correspond to the average conditions that existed on the shores of the lakes, at all four stations, during the particular time of the year in which the data were being obtained. The experiments indicate that the effect of the wind on evaporation is fundamentally different from that indicated by the common evaporation formula. The rate of evaporation from open-water surfaces when the air temperature is below freezing has been investigated for the first time.

Introduction

The physical conditions that affect evaporation on the Great Lakes are quite different from conditions in comparatively shallow lakes. Because of the great depths involved (Lake Superior being as much as 1 300 ft deep in parts, Lake Michigan 920 ft, Lake Huron 750 ft, Lake Erie 210 ft, and Lake Ontario 760 ft), the water temperatures tend to remain relatively constant. Measurements of water temperatures in Lake Superior indicate that the entire body, below a depth of 180 ft, remains at a temperature of approximately 39.4° F throughout the year, which is the temperature of maximum density. In summer, the upper water is at a higher temperature than the lower body of water; and, in general, the temperature becomes increasingly warmer from a depth of 180 ft to the surface. In winter, the upper water is colder than the lower body;

[6] Junior Engr., U. S. Lake Survey, Detroit, Mich.

and, in general, the temperature becomes increasingly colder from the lower depths to the surface.

The average wind velocity on the lakes is about 10 miles per hr, which is the 15 yr average for four stations located around Lake Superior, the stations being about 100 ft above the lake surface where the wind velocity is comparable to that on the lake. There is considerable evidence to indicate that the effect of the wind is to cause a circulation and interchange of water between the upper and lower layers of the lake, and that there is some interchange most of the time. The effect of this interchange is to keep the surface temperature, which affects evaporation, cooler in spring and summer than would be the case in a shallow body of water; and, the water temperature lags considerably behind the air temperature in the spring and summer when the air temperature is rising. There is a similar lag in the water temperature in the fall and early winter when the air temperature is decreasing. As far as is known, none of the lakes freezes entirely over in the winter, although such a condition might be possible for a short time.

John R. Freeman, Past-President and Hon. M. Am. Soc. C. E., and others who have studied the subject have made attempts to deduce the lake evaporation from the standard evaporation formulas. Realizing that his results were not satisfactory, Mr. Freeman stated[7] that, "Evaporation is at least a matter of great scientific interest, and a precise knowledge of the causes leading to change in its amount could be of use * * *. The collection of accurate statistics can not be begun too soon." After considerable study, he recommended the following procedure for the collection of data:

(1) A few months of experimental work, with relatively inexpensive equipment, for obtaining accurate values of constants in the formulas, by means of which evaporation can be estimated from standard records of the U. S. Weather Bureau Stations, which give the temperature, dew point, and velocity movement of the air.

(2) At each lake, one, or preferably two, evaporation tanks, on opposite sides (one in Canada and one in the United States) each preferably 6 ft or 8 ft in diameter by 4 ft to 5 ft in depth, of rigid shape, set on a rigid support high enough above the lakes to escape waves, and located on some projecting pier or on the shore. This type of tank should have provision for warming the water in winter barely enough to keep the ice from freezing on the surface, should have a device for agitating the surface slightly, and an anemometer should be set beside it.

(3) One daily temperature measurement shown by the maximum-minimum thermometers should be made along with observations for all the meteorological factors affecting the evaporation.

(4) Comparison should be made between the wind velocities at the pan and those recorded at the regular Weather Bureau Station in the same vicinity.

Because present formulas were not satisfactory for the required determination, new experiments which were more or less in accord with Mr. Freeman's recommendations were undertaken.

[7] "Regulation of the Great Lakes," by J. R. Freeman, p. 421.

Description of Equipment

All stations were located near the lake shore away from any buildings or land obstructions which might create atmospheric disturbances. The design and arrangement of the equipment approximated that used by the Weather Bureau as nearly as possible in order to facilitate comparison of data. The dimensions of the pan and relative positions of the instruments were identical. However, the pan had several new features which heretofore had never been used.

Design of Pan.—In order to control and regulate the temperature of the water in the pan, heat had to be supplied in winter and refrigeration in summer. The original design, shown in Fig. 3, called for a water-jacket around the pan

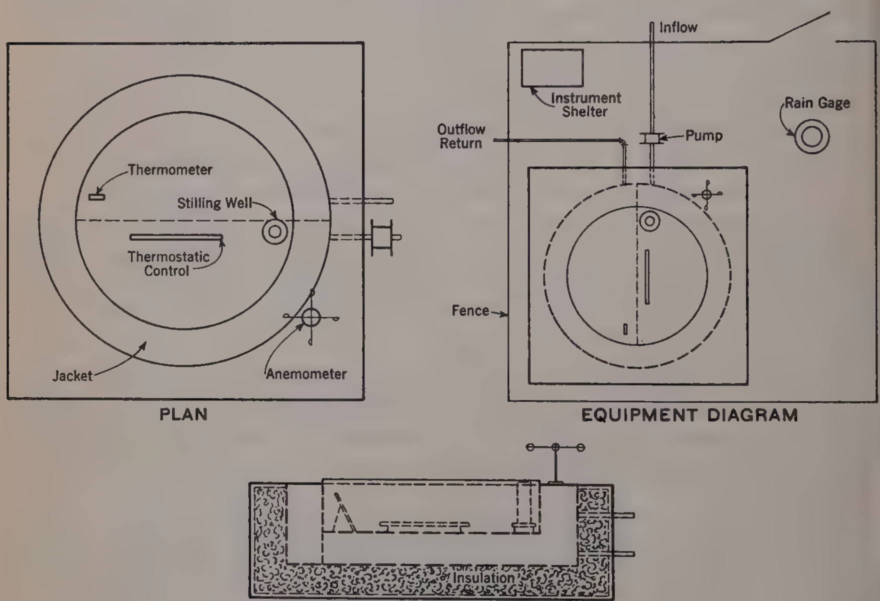

Fig. 3.—Design of Detroit Evaporation Unit

where warm or cool water could be circulated. This design was modified in two cases because of local conditions. At Duluth, enclosed electrical heating elements were submerged in the pan water; and at Detroit, electrical heating elements were placed in the de-watered jacket. The supply of heat was regulated by thermostatic control. All units were cooled in summer by circulating cold water around the tank. The supply of cooling water was regulated periodically, by hand, as the need arose. The source of supply was either cold well-water or water pumped from the lake at a depth of 20 ft or more. The pans were placed in large insulated boxes which aided in the regulation of the temperature.

It has been mentioned that an attempt was made to keep the temperature of the pan water as nearly as possible to mid-lake temperatures at all times.

Due to mechanical difficulties, the control was far from perfect; but the results were entirely satisfactory when analyzed by the method which was used and which does not require that the pan water be at the same temperature as the lake water.

Average Daily Temperature.—A maximum-minimum thermometer was placed in the pan water to obtain the mean daily temperature. It was assumed that for all practical purposes the mean of the maximum and the minimum water temperature for the 24-hr period represented the average temperature for the period. As a check on this assumption, observations every half hour for 24 consecutive hours were made on a number of different days. The thermometer was also read for each of these different days. The average of the 48 half-hour readings for each day, in all cases, checked the mean of the maximum-minimum value for the same day within 0.5 degree.

The method of obtaining the average daily air temperature was the same as that used by the U. S. Weather Bureau. The mean of the maximum and minimum thermometers gives the average temperatures, accurately for the day, except in rare instances.

Other Equipment.—The total daily movement of wind over the pan was measured by a standard anemometer placed at the edge of the pan and several inches above the water surface. These daily values were reduced to miles per hour before being used in the analysis. Rain and snow gages were placed around the pan to record precipitation. Wet-bulb and dry-bulb thermometers were supplied to all observers for daily observations of relative humidity and only one reading a day was taken in the immediate vicinity of the station.

Results of Experiments

Because of the similarity of the equipment and the method of obtaining the daily readings, it can be assumed that mechanical inconsistencies and personal errors were reduced to a minimum. For purposes of analysis, all daily observations were grouped according to water temperatures, air temperatures, and wind velocities, regardless of location, it being assumed that the relative humidity and barometric pressure were normal for the particular temperature combination under consideration. For certain temperature combinations, this may not be true; but if a sufficient number of observations are taken within any one range, the average relative humidity value for that range will approach normal. The accuracy of a statistical analysis of this nature depends on the number of observations.

Master Chart.—The daily readings, listed according to temperatures and wind velocities, were arranged again into groups with a water and air temperature range of 5 degrees. For purposes of illustration, one group is described briefly: 44 daily readings—11 from the Duluth station, 14 from Kewaunee, 10 from Detroit, and 9 from the Buffalo station—where the average daily air temperature of each reading was between 40° and 45° and where the average daily water temperature was between 45° and 50°, were included in this group. The centroid of these 44 readings was found and plotted, and the average of these readings was assigned this point. The average wind velocity of the group of readings was found to be 5 miles per hr. Approximately 300 daily readings

from the four stations were grouped, averaged, and plotted in this manner. An additional 300 observations daily, scattered throughout the period of the experiments, were not used. These 300 readings represented days during which there was considerable rain or snow accompanied by high winds. Under these conditions, the evaporation pan did not collect the same amount of precipitation as the rain gage, and although individual readings of the evaporation pan "catch" were sometimes high and at other times low, it was decided not to include these irregular observations in the averages because of these discrepancies.

Some groups had as many as eighty individual readings and others had only five. The average wind velocities in the various groups or squares ranged from 4 to 6 miles per hr, with most of the squares averaging 5 miles per hr. Each individual square was studied in order to determine the effect of the wind on evaporation in that square. It was found that the increase in the amount, due to an increase in the wind velocity, did not vary in the same proportion throughout the range of the experiments. The proportion in each square was different and followed a well-defined plan. The value of average evaporation, in those squares where the average wind velocity was either greater or less than 5 miles per hr, were decreased or increased, according to the proportion for the respective square, to correspond to a value of evaporation based on a 5-mile wind.

Contours representing equal lines of evaporation were drawn. Fig. 4(*a*) is the final result of this work representing inches of evaporation per month instead of inches of evaporation per day. Daily values may be obtained by dividing by 30. The curves are based on a wind velocity, measured at one foot above the water surface, of 5 miles per hr. It has been assumed that relative humidity and barometric pressure correspond to the average conditions that existed during the particular period of the year in which the data were being obtained at the four stations. In order to determine whether discrepancies, due to location or methods, existed in the data, charts for each station were made. No irregularities in the slope or position of the contour lines were evident. Fig. 4(*b*) represents values of monthly evaporation with a wind velocity of 10 miles per hr, and Fig. 4(*c*) represents values of monthly evaporation with zero wind velocity according to the formula[8] proposed by Adolph F. Meyer, M. Am. Soc. C. E.

$$E = 30\,(V - v)(0.5 + 0.05\,W) \qquad (4)$$

Temperature Loop.—The range of the two master charts (Figs. 4(*a*) and 4(*b*)) is sufficient to cover average conditions on the Great Lakes, and monthly values of evaporation may be obtained directly from them if the respective mean monthly air and water temperature and wind velocity data are available. It is known that the air temperature for individual days drops down well below the low point of the charts. Should it be necessary to obtain values for extreme ranges in temperature or wind velocities, the curves may be extended; daily observations under extreme conditions have been plotted, and they checked the extended contours very well.

To obtain the monthly values of open-water evaporation for any lake, it is necessary to have accurate information as to the air temperatures over the lake,

[8] "Elements of Hydrology," by A. F. Meyer, p. 238.

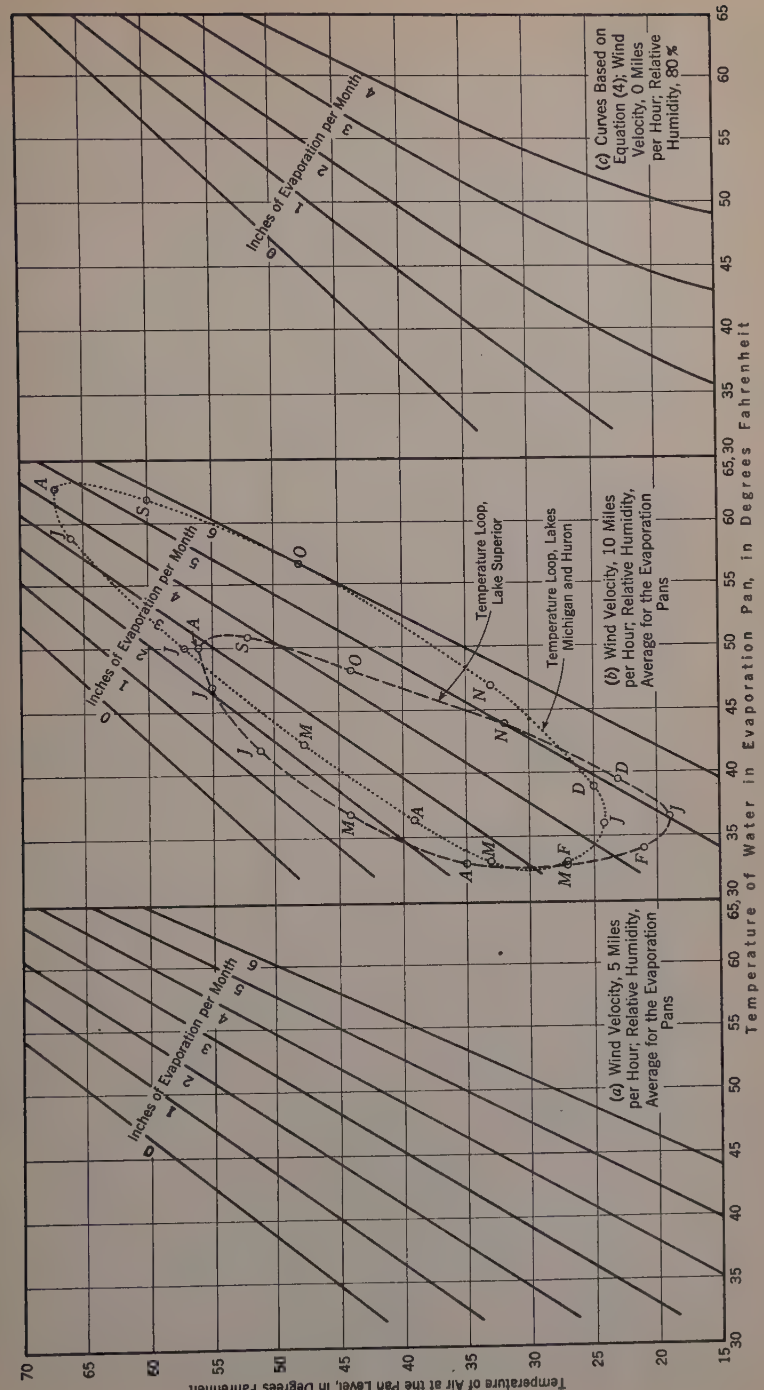

FIG. 4.—MONTHLY EVAPORATION CHARTS

the average water temperatures, and the average wind velocity over the lake. Considerable attention was given to these factors. Mean monthly air temperatures obtained from the records of the U. S. Weather Bureau Stations along the shore of the lakes do not represent mid-lake conditions. Readings observed out on the lake differ materially from those taken on shore because the air is influenced by the temperature of the water, both in summer and in winter.

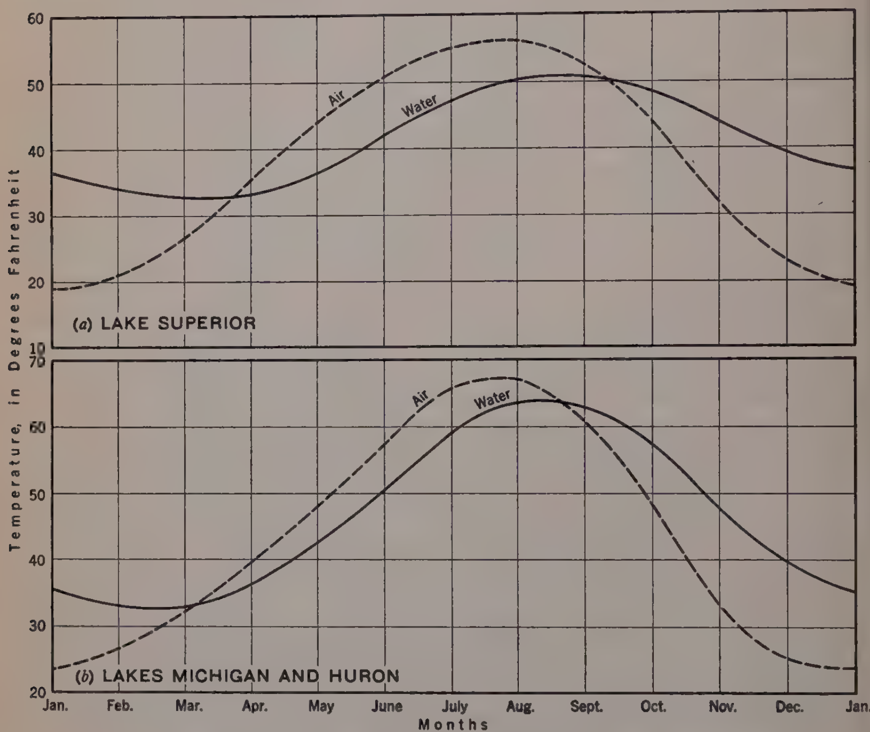

FIG. 5.—AVERAGE MEAN MONTHLY VALUES OF AIR AND WATER TEMPERATURES, WITH OPEN-WATER CONDITIONS

The mean monthly air temperatures as shown in Fig. 5 represent average values deduced from readings observed at the middle of the lake. Information pertaining to temperatures, both water and air, was obtained from the records of Lake Survey steamers, lake freighters, ferry boats, lighthouses, and municipal water-supply departments.

Seasonal air temperature variations in the lake region are considerable, and although these variations are somewhat decreased by the lakes, they do affect the temperature of the water. Large bodies of water, and particularly Lake Superior, react slowly to climatic variations in temperature. In fact, the yearly range of the average mean water temperature as shown in Fig. 5(*a*) is only 18 degrees. Due to the size and depth of the lakes, it can readily be seen that a large amount of heat is necessary to produce temperature changes in the water. The lake is never at the same temperature throughout, at any one time,

and the temperature in the middle may be 20° lower than that recorded near the shore. No matter how the air temperature varies, and at what rate, the temperature of the surface water lags behind that of the air. The mean monthly values for lakes Superior, Michigan, and Huron, shown in Fig. 5, represent average open-water conditions.

The surface water is subject to frequent changes in temperature when there is no wind; but since this condition seldom exists, interchange of surface and sub-surface water is occurring continually. The wind is one of the most important factors to be considered in the study of lake evaporation for this reason.

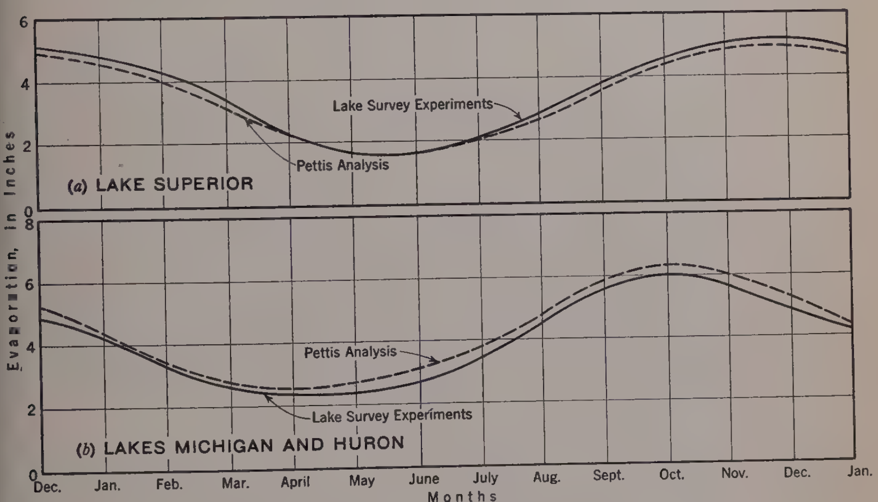

FIG. 6.—EVAPORATION FROM THE LAKE SURFACE

In strong winds, spray is thrown into the air, thus increasing the evaporation. Observations at Weather Bureau Stations on land indicate an average wind velocity of 10 miles per hr and it is believed that the velocity over the water is at least as great.

If the normal mean monthly air and water temperatures for Lake Superior, as shown in Fig. 5(*a*), are plotted in Fig. 4(*b*), the points will form the broken-line loop shown. These normal monthly values are plotted in Fig. 6(*a*) and are compared with the values of normal monthly evaporation obtained by an entirely different analysis made by Colonel Pettis. It was for this comparison that the study was made. A temperature loop and values of monthly evaporation for lakes Michigan and Huron are shown in Fig. 4(*b*) and by the dotted loop in Fig. 6(*b*).

FACTORS AFFECTING EVAPORATION

Evaporation from the lakes, as shown by the charts, has been related to air and water temperature and wind velocity. The evaporation, obtained as a result of the experiments, cannot be related directly to relative humidity or barometric pressure as measured at standard Weather Bureau Stations. The

temperature of the air and water influencing evaporation, and likewise the velocity of the wind, may be used with considerable accuracy, but no correlation could be found between evaporation and relative humidity. No evidence could be found that the amount of evaporation varied with either the relative humidity as read at the station or as determined at neighboring Weather Bureau Stations. Although both relative humidity and barometric pressure have not been related directly to evaporation, they have been considered. It is logical to assume that if a sufficient number of observations are made in the same locality, over a sufficiently long period of time, using a particular air-and-water temperature combination, the resulting average relative humidity and barometric pressure will be characteristic for that temperature combination, for that locality.

In other words, evaporation for any month, say September, as obtained from Fig. 4(*b*), is the most probable value of the evaporation for that month, under normal or average conditions of relative humidity for that particular month. This is true since the evaporation curves for this particular locality were obtained from numerous daily readings of evaporation, taken under conditions of relative humidity which existed in this region, in this particular season. Fig. 4(*b*) thus gives the results in the most desirable form to use in any study, either general or specific, which involves average or probable conditions.

In order to emphasize the reason for basing this analysis on the three factors, air and water temperature and wind velocities, it might be well to analyze the ordinary evaporation formula that has been in use for some time.[9] Considering the equation as a modification of Dalton's Law

$$E = K\,(V - v)\left(1 + \frac{w}{c}\right) \qquad (5)$$

in which K = constant for particular conditions; w = mean velocity of wind in miles per hour; and c = coefficient expressing the effect of wind. It will be noticed that the measure of evaporation is the difference between V and v, multiplied by a constant and a wind correction. In other words, the measure of evaporation expressed by Equation (5) depends upon the saturation deficit of the atmosphere. If the saturation deficit is small, the evaporation is small. Should a condition exist in which the water temperature on Lake Superior was 40° and the air 42.4°, with a wind velocity of 10 miles an hr and a relative humidity of 80%, as measured by the Weather Bureau, a condition quite common for this lake, the evaporation for the day would be zero, according to Equation (5). The same temperature and wind combination on the master chart (Fig. 4(*b*)) would give 0.09 in. of evaporation for the day.

In order to use Equation (5), the temperature of the water and air must be known. After reducing the air temperature to the dew-point temperature, dependent on the relative humidity, each is converted into pressures. It is assumed that the moisture in the air influences the rate of evaporation. This cannot be denied, but experiments indicate that under the influence of wind, evaporation occurs consistently, although Equation (5), used with the proper climatic factor, indicates that the evaporation would be zero. Evaporation

[9] "Regulation of the Great Lakes," by J. R. Freeman, 1926 Edition, p. 135.

increases relative humidity, and it is believed that relative humidity, as indicated by the experiments, is a result of evaporation more than a cause of evaporation.

Experiments conducted in different parts of the United States by the U. S. Weather Bureau show evidence that a humidity gradient exists, varying inversely as the elevation above the earth's surface. Even if the relative humidity is 100% at the water surface, there is atmosphere directly above where the

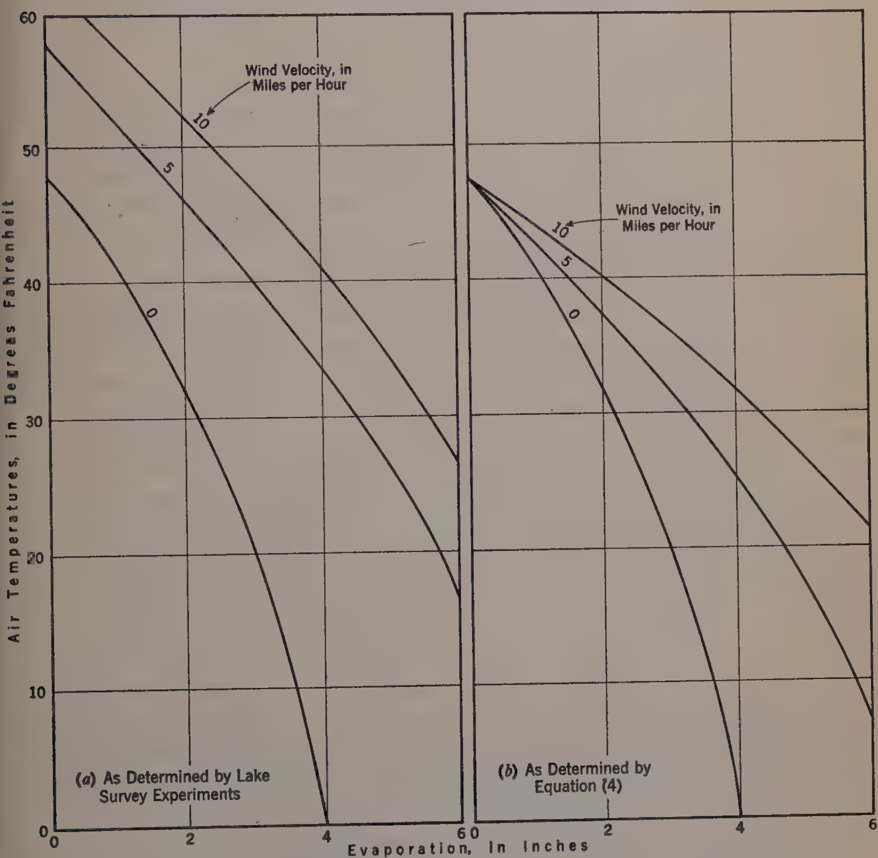

Fig. 7.—Values of Monthly Evaporation for a Constant Water Temperature of 45 Degrees Fahrenheit

percentage is less, and there is a continual escape of vapor upwards. It seems logical to assume, for the purposes of this presentation, that evaporation is dependent on temperatures and wind velocities, and that relative humidity on the Great Lakes tends to follow the seasonal averages.

If the wind factor in Equation (5) is considered, it will be noticed at once that E approaches zero as the difference between the vapor tension and the vapor pressure becomes less. When the values of V and v are equal, evaporation ceases, regardless of the wind velocity. Experiments described in this paper

show that this is not true. The air temperature is higher for zero evaporation, assuming a constant water temperature, as the wind velocity is increased (see Fig. 7), and similarly the water temperature is lower for zero evaporation, assuming a constant air temperature, as the wind velocity is increased. According to Mr. Meyer,[8] an increase in the wind velocity from zero miles per hr to 10 miles per hr, assuming that other factors in Equation (4) remain constant, doubles the evaporation. With a constant water temperature of 45° F and an air temperature of 32° F, the monthly value of evaporation increases from 3 in. to 4 in., as the wind velocities increase from 5 to 10 miles per hr, relative humidity remaining constant. Similarly at 25° F, the monthly value of evaporation increases from 4 in. to 5.3 in., or an increase in both cases of 33% (see Fig. 7(*b*)). If the same temperature combinations are used with Fig. 7(*a*), it will be noticed that the increase of monthly evaporation, due to an increase in the wind velocity from 5 to 10 miles per hr, for an air temperature of 32° F, will be from 4.2 in. to 5.3 in., or an increase of 26%, whereas the increase at an air temperature of 25° F, for the same conditions, will be from 5.0 in. to 6.1 in., or an average increase of 22 per cent.

At 48° F, the increase, due to an increase in the wind velocity from 5 to 10 miles per hr, according to Fig. 7(*b*), is zero. At this temperature, Fig. 7(*a*) gives an increase in the values of monthly evaporation, with an increase in wind velocity from 5 to 10 miles per hr, of 80 per cent. The wind has been found to have a decided effect on evaporation, particularly when the air and water temperatures are nearly equal. The comparison with the Meyer formula is as accurate as could be made with the data available. The evidence indicates that current opinion as to the effect of wind on evaporation is in need of revision.

With present-day mechanical equipment, much could be learned about the laws of evaporation if tests could be made in a wind tunnel under laboratory conditions. With little effort and expense, all factors governing evaporation could be controlled, each could be isolated and its effect determined.

Acknowledgments

The data from which the air and water temperature curves were prepared were obtained from the files of the U. S. Lake Survey Office and from information personally obtained by the writer from a number of different sources. Copies of the monthly evaporation reports are on file at the various sectional headquarters of the U. S. Weather Bureau. Mr. Freeman's report[7] on "The Hydrology of the Great Lakes" was consulted frequently.

The data form a part of a thesis[10] by the writer, entitled "Evaporation from the Great Lakes," presented to Lawrence Institute of Technology, Highland Park, Mich., in 1938, in partial fulfillment of the requirements for the degree of Doctor of Engineering. The writer wishes to acknowledge the invaluable assistance of Colonel Pettis and Mr. Sherman Moore, of the Lake Survey, Professors H. L. Woolfenden and G. P. Brewington of Lawrence Institute of Technology; Professor C. W. Chamberlin of Michigan State College, at East Lansing, Mich.; and valuable constructive criticism of Professors C. O. Wisler and W. C. Hoad of the University of Michigan, at Ann Arbor.

[10] A copy of the thesis, containing all observational data, has been filed for reference in the Engineering Societies Library, 33 W. 39th Street, New York, N. Y.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

DESIGN OF A HIGH-HEAD SIPHON SPILLWAY

By Elmer Rock,[1] Jun. Am. Soc. C. E.

Synopsis

A direct and clear-cut method of analysis, for use in determining the proportions of a high-head siphon spillway, is presented in this paper. The manner in which well-established principles of hydraulics are applied is outlined, general equations for the determination of throat area and outlet area are derived, and the application of the method as presented is illustrated by the solution of a specific problem.

Introduction

The siphon spillway has become increasingly popular as a device for the automatic control of water-surface elevations. In reservoirs maintained for recreational purposes, in canals, in head-water bays of hydro-electric power plants or, in general, wherever close regulation of pool elevation is necessary, such a device can be used to distinct advantage. Numerous installations in the United States and elsewhere have proved their practicability and adaptability, and they indicate that the siphon spillway occupies a definite place among worthy hydraulic structures.

This paper applies, in the main, to problems arising in connection with the determination of the proportions of a high-head siphon spillway. The term "high-head siphon" is used by the writer to denote siphons that operate under heads in excess of 34 ft, or the barometric height. The head in this case is the difference in elevation of the head-water and tail-water surfaces, or, if the outlet discharges into atmosphere, it is the difference in elevation of the head-water surface and the outlet. Siphons operating under heads less than 34 feet are termed "low-head siphons." The barometric height in feet of water represents the magnitude of atmospheric pressure and is of paramount importance when the action of high-head siphons is considered. The manner in which its value enters into design computations and the effect it has upon siphonic action will be treated subsequently herein.

Strictly, the head at which a siphon becomes a high-head siphon should include some mention of head losses within the siphon tube as well as practical

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limitations on working pressures. For example, let it be assumed that the total available head at a site is 32 ft, the head lost in the entire structure is 6 ft, and the allowable negative pressure is 24 ft of water. This would be a high-head siphon because there is an excess of head (2 ft) over and above the amount it is possible to utilize for producing flow. Cognizance of this excess head comprises one of the special considerations dealt with in this paper. The evaluation of head losses and the selection of an allowable negative pressure are given further mention in a later paragraph. It is believed that, because of its common usage and its vital significance, 34 ft serves, appropriately, to designate the dividing line between a high-head and a low-head siphon. Many of the principles contained in this paper may be applied to low-head siphons with equal effectiveness. The conditions dealt with in the design of low-head siphons, however, present separate problems of a different nature and no attempt will be made to include siphons in that classification.

In the past there has been some discussion concerning the feasibility and the performance of high-head siphon spillways. In general, it is now definitely established that heads far in excess of 34 ft may be utilized, and the action of spillways designed to function under such heads can be predicted with a reasonable degree of accuracy. It is probable that, in some cases, the tendency to over-design a siphon spillway is present. In other words, a factor of ignorance is included to offset the lack of a dependable basis for design. This is true because access to a definite rational method of analysis, carried through from beginning to end, and pertaining to the case at hand, is difficult to find in American literature. A review of the published articles dealing directly with this subject will reveal that methods presented are generally indirect and clumsy.

Some methods are based on the formula

$$Q = C\,A\,\sqrt{2\,g\,H} \qquad (1)$$

in which: Q = the discharge; C = a coefficient of flow; A = area of cross-section; g = acceleration due to gravity; and, H = total hydraulic head available. Experiments and prototype measurement generally establish the value of C between 50 and 80 per cent. Equation (1) serves well to establish a basis for comparison of siphon spillways. It is inadequate for design purposes, however. In any case, it necessitates a cut-and-try procedure and does not take into account local conditions at critical sections. Particularly in the design of high-head siphons, it is inadequate in that no provision is made to convert the excess head into some form of energy such as heat or kinetic energy. The writer proposes to show how excess head is of utmost importance, and how it is provided for by proper dimensioning of the lower leg. In view of these shortcomings it may be said that, as it stands, Equation (1) does not represent a direct and thorough approach. Methods based on a study of the hydraulic gradient alone fail to take into account the nature of the flow in the upper bend where velocity distribution is entirely different from that in a straight pipe. The velocities used in the computation of this gradient are mean values and the pressures at the throat section, as measured from the

gradient, do not represent actual localized pressure intensities. The writer does not wish to minimize the value of a plot showing the hydraulic gradient; certainly, it can be used to a great advantage, but its limitations must be recognized.

Upper Bend and Throat

At the outset it must be understood that when a siphon is operating under a head greater than 34 ft, the excess head cannot be used to increase flow through the spillway. If two siphons were constructed, each having the same throat area and designed for maximum discharge, one operating under a 34-ft head and the other operating under an 80 ft head, the latter would not have a greater capacity. This fact is easily understood when the fundamental principle of siphonic action is considered. The lower leg of a siphon makes possible the existence of extremely low absolute pressures in the upper bend, but the only force being exerted to produce flow is the pressure of atmosphere on the head-water surface. Obviously the magnitude of this force is the limitation imposed by nature on all siphons.

Generally, the critical section of any siphon is at the crest. At this section the negative pressures are of greater intensity than at any other section in the siphon tube. Hence it may be concluded that the dimensional characteristics of the structure at its summit determine its capacity. In design work, therefore, it is only natural for initial studies to be confined to the throat of the siphon.

The upper bend is usually formed with regular curves of moderate radii. Almost always the tube is rectangular in cross-section at the summit. The manner in which the water flows—that is, its velocity distribution when passing through this portion of the siphon tube—is the basis for determining the size of the throat opening. The most recent experimental data show conclusively that when water flows through a bend in a conduit the velocity distribution is not symmetrical about the axis of the conduit cross-section. Furthermore, they indicate that velocities are greatest at the inside of the bend and decrease in magnitude toward the outside of the bend. The condition of flow is comparable to that in a free vortex. In the case of a siphon where the upper bend is a rectangular conduit it may be said that, for all practical purposes, the condition is analogous to free vortex flow. It is possible then to utilize the laws governing vortex action in the design of the throat section.[2]

Before any determination of proportions can proceed it is necessary to adopt a minimum absolute pressure that can be tolerated in practice. Theoretically, a high-head siphon could be designed to develop a vacuum of 34 ft of water at its summit, and this would represent the greatest possible efficiency. In other words the absolute pressure would be zero, or the lower theoretical limit. In practice, such degree of rarefaction can never be attained; vapor pressure and elevation above sea level decrease this figure appreciably. Water in its natural state has air in solution and, as the absolute pressure decreases, the quantity of air drawn from the solution increases. In the case of the siphon, this action may cause sufficient air to collect at the summit to

[2] "Siphon Spillways," by A. H. Naylor, published in London, England, 1935, by Edward Arnold.

interrupt the flow. Furthermore, with negative pressures approaching 34 ft of water, it would not be unreasonable to expect parting of the water from the crest, with its accompanying vibration and cavitation. As a general rule experience with siphon spillways has established 24 ft of water as a maximum negative pressure to be allowed at any point in the structure.[3] In other words the absolute pressure should not be permitted to be reduced below 10 ft of water.

Frequently many of the dimensional characteristics of a siphon spillway are fixed by physical limitations of the structure in which it is incorporated. Thus the radii of curves which form the upper bend may not be dictated by consideration of hydraulic performance. If such limitation does not exist, then two general rules may be applied: (1) As the radius of curvature is increased the cross-sectional area to be provided at the throat decreases; hence it is obvious that excessively sharp curves should be avoided (the logic in this statement is demonstrated subsequently herein); and (2) as the radius of curvature decreases the priming qualities are enhanced. Thus to forestall difficulties from objectionable priming action it is desirable to avoid unduly large radii.

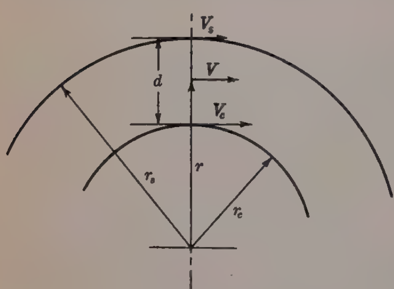

FIG. 1

Having thus fixed the minimum permissible absolute pressure and the upper bend radii, which tie in the depth of the throat section, it is possible to determine the discharge per foot of spillway crest. The maximum head to be utilized for producing velocity is 24 ft. Then the velocity in the upper bend cannot exceed $\sqrt{2\,g\,H} = \sqrt{2 \times 32.2 \times 24} = 39.3$ ft per sec. Referring to Fig. (1), the following procedure is established:

In free vortex flow................	$V \times r = \text{constant}$	
Hence..........................	$V \times r = V_c \times r_c$	$= V_s \times r_s$
Or..............................	$V = \dfrac{V_c\, r_c}{r}$	
From the foregoing computation....	$V_c = 39.3$ ft per sec	
Then............................	$V = \dfrac{39.3\, r_c}{r}$	

The discharge for a small element of depth dr is

$$dQ_1 = V\,dr \qquad (2)$$

Substituting for V

$$dQ_1 = \frac{39.3\, r_c}{r}\,dr \qquad (3)$$

[3] *Proceedings*, Inst. C. E., Vol. 231 (1930–31), p. 201.

Integrating between the limits of r_c and r_s

$$Q_1 = 39.3\, r_c \int_{r_c}^{r_s} \frac{1}{r}\, dr = 39.3\, r_c \log_e \frac{r_s}{r_c} \qquad (4)$$

Equation (4) is an expression, in terms of the upper-bend radii, for the maximum discharge per foot of crest that can obtain without exceeding the criterion for minimum absolute pressure. If Q represents the total discharge for which the siphon is being designed, the length of crest, or width of throat section, is given by

$$B = \frac{Q}{Q_1} \qquad (5)$$

The throat area to be provided is thus determined and equal to the width times the depth. Equations (4) and (5) are perfectly general and can be applied to the upper bend of any spillway whether it is a high-head or a low-head siphon. They indicate the discharge that can pass safely through the upper bend. The remainder of the siphon tube must be proportioned in such a manner that this discharge is not exceeded.

Inlet

The next step is to provide a well-proportioned inlet. There is no "hard and fast" analytical method of procedure that can be followed in the design of this opening. As is the case in any conduit intake, it is desirable to keep the entrance velocity low and to avoid sharp corners where flow lines are curved. The inlet should be sufficiently removed from the head-water level to prevent the siphon from sucking air through the surface depression caused by the formation of a free vortex in the head-water bay. With low entrance velocities, however, the probability of having a pronounced vortex is lessened. Hydraulic considerations and experience with siphons have indicated that entrance losses are minimized when the inlet area is two or three times the area of the throat and is formed with well-rounded corners. To prevent the entrance of ice and drift, and to guard against surface disturbances, it is desirable to locate the opening in such a manner that it is at all times well below the water surface. Usually the inlet is protected by rack bars spaced quite far apart.

Upper Leg

The design of this part of the siphon tube generally presents little difficulty. Probably the only requirement is that the upper leg should taper gradually from the inlet to the throat, thereby providing a smooth transition in which acceleration is affected.

Lower Leg

Primarily the function of the lower leg of a siphon is to make possible the existence of the partial vacuum that must occur at the summit, and to provide a channel through which the water may escape and be directed to the point of discharge. In the case of a low-head siphon no especial difficulty need be experienced with the lower leg. It is simply a matter of making the tube

uniform and equal in area to the throat section, and positive action can be expected. Frequently the lower leg is constructed to incorporate a gradual expansion in cross-sectional area thereby making use of draft action to increase the discharge. When the capacity of the spillway has been determined, the pressures developed at the summit should be investigated by application of Equation (4). If such analysis indicates that the absolute pressure is below the practical limit (say 10 ft), the upper bend should be redesigned to prevent the existence of undesirable negative pressures. For a high-head siphon, however, the problem becomes more involved and warrants a careful analysis. In a high-head siphon, if the lower leg were to be made equal in cross-sectional area to the throat section, and made uniform throughout, excessive vacuums would prevail. Air would be drawn from solution, cavitation would occur at points of reduced pressure, and parting of the water from the siphon tube could be expected. As a result siphonic action would be interrupted intermittently, with accompanying vibration and violent shock.

As has been explained previously, the maximum head that can be utilized to produce flow is 24 ft, plus the head lost due to friction, bends, entrance, etc. Consequently the total head available in excess of that amount cannot be used to increase the discharge of the siphon. That excess head may be expended in two ways: (1) A part of it can be dissipated by introducing resistances such as a bend and friction; and (2) the remainder can be converted into kinetic energy by increasing the velocity of flow at the outlet; that is, by reducing the cross-sectional area of the siphon tube near the outlet by means of a reducer. In effect these resistances provide sufficient back pressure in the lower leg of the siphon to keep the tube flowing full. The problem then is to determine the necessary reduction in outlet size to accomplish this without restricting the capacity of the throat opening.

The cross-sectional area of the throat, the upper bend radii, and the upper leg proportions have already been fixed. To facilitate the following derivation the lower leg is assumed to be equal in area to the throat opening and of uniform cross-section throughout. In this manner the problem is reduced to one that lends itself to analytical solution.

It is possible, now, to compute the head loss in the siphon tube for the discharge Q. The losses should include those due to entrance, friction, bends, and the discharge on exit loss. This determination involves merely a selection of head-loss coefficients and does not warrant further comment. All of the losses can be expressed as a percentage of the velocity head (h_v) in the throat section. Hence, if $K_1 h_v$ is the head loss due to the entrance, friction, bends, etc., and, $K_e h_v$ is the head converted into kinetic energy, then the total head lost is $K_1 h_v + K_e h_v$ or, $h_v (K_1 + K_e)$. It must be remembered that in this expression if no reducer were provided the value of K_e would be equal to unity. Assuming the siphon to discharge into atmosphere, it is evident that the total head lost must equal the total head available. In other words the hydraulic gradient must intersect the siphon tube at its outlet; thus:

$$H = (K_1 + K_e) h_v \quad \text{.......................(6)}$$

The exit loss coefficient may be expressed in terms of the reference area (the area A_t, at the throat) and the area of the reduced outlet, A_r; that is:

$$K_e = \left(\frac{A_t}{A_r}\right)^2 \qquad (7)$$

Substituting the value for K_e (Equation (7)) in Equation (6):

$$H = \left[K_1 + \left(\frac{A_t}{A_r}\right)^2\right] h_v \qquad (8)$$

and, solving for A_r:

$$A_r = A_t \sqrt{\frac{h_v}{(H - h_v K_1)}} \qquad (9)$$

Thus, the area to which it is necessary to reduce the siphon tube at the outlet can be expressed in terms of quantities given in the basic data or determined from the foregoing calculation. By use of Equations (4), (5), and (9) the proportions of a high-head siphon spillway can be determined.

Solution of a Specific Problem

In order to clarify the application of the method presented herein the following concrete example is solved completely. The basic data chosen are: (1) The discharge for which the siphon is to be designed is 350 cu ft per sec; (2) the outlet is to discharge into atmosphere; (3) the head available is 80 ft; (4) the permissible negative pressure is 24 ft of water; and (5) to compute the head lost due to frictional resistance a value of 0.012 is used for Kutter's n (this is a fair value in practice for a tube lined with cast iron). Detailed mention of the individual head losses is believed unnecessary. In the selection of head-loss coefficients, it is interesting to note that even sizable errors (for instance, using an n value of 0.020 instead of 0.012, for friction loss) have but little effect upon the hydraulic performance. If the coefficients used are too large, the actual discharge and velocities realized will be slightly greater than computed, thus increasing the total head lost. These effects are interrelated and tend to offset each other, with the result that a balance is reached at a discharge not far in excess of the original design value. A similar readjustment will occur if the coefficients used in the design computations are smaller than the actual values in the finished structure. Let it be assumed that consideration of the priming qualities desired and the physical limitations of the structure, as well as other factors, have fixed the radii of the upper bend as 4 ft and 7 ft. The problem is to determine the proportions of the siphon spillway so that the required capacity is obtained with satisfactory hydraulic performance. Fig. 2 is a graphical representation of the basic data selected. The form of the siphon tube was chosen at random and does not follow the pattern of any existing structure.

The first step is to determine the size of throat opening necessary so as not to exceed practical limitations on the minimum permissible absolute pressure at the crest. Equations (4) and (5) are derived in such a manner that the throat dimensions obtained by their application will conform to the fore-

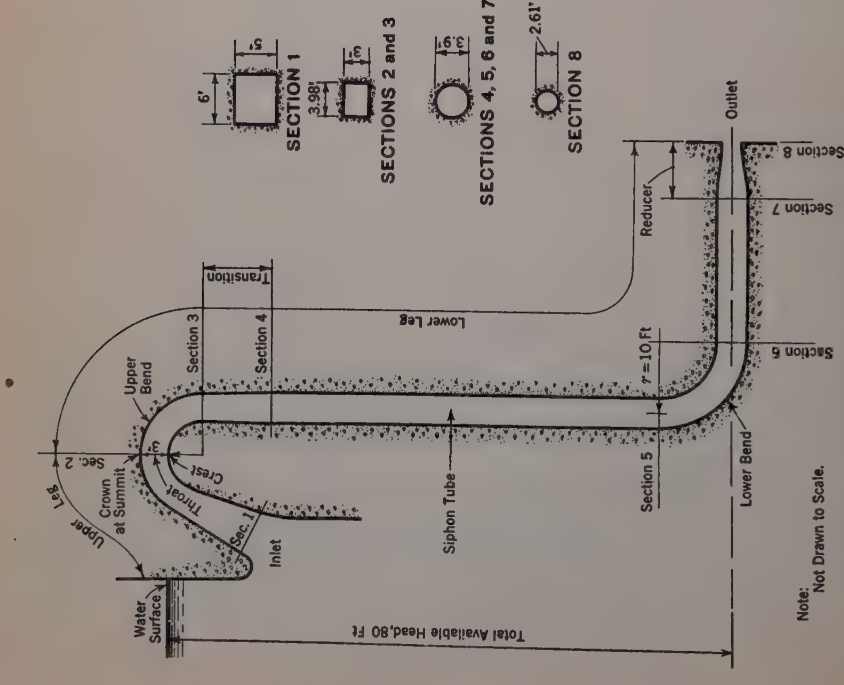

FIG. 2.—GENERAL LAYOUT SHOWING PROPORTIONS AND TERMINOLOGY

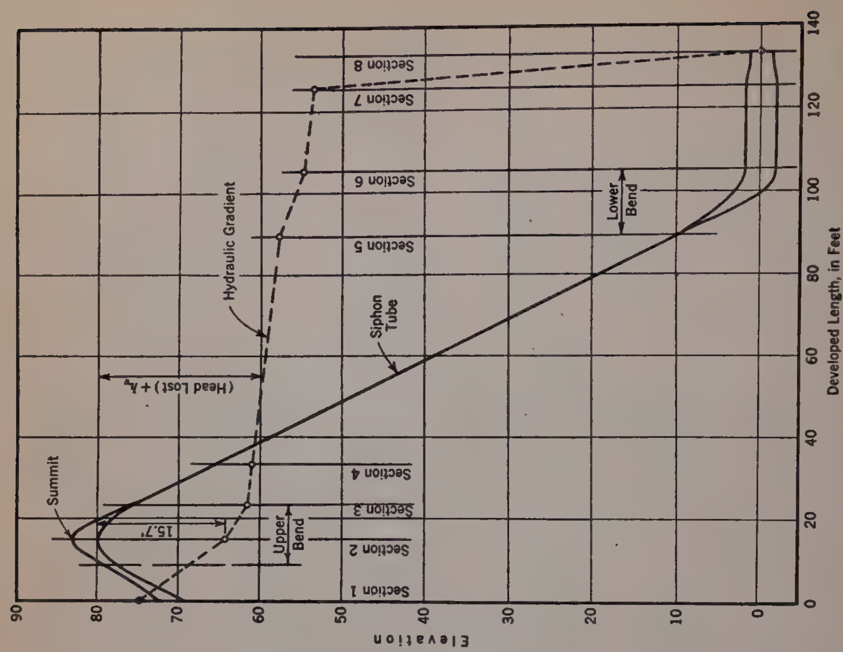

FIG. 3.—PLOT OF COMPUTED HYDRAULIC GRADIENT (DISCHARGE, 350 CUBIC FEET PER SECOND; AND POOL ELEVATION 80 FEET)

going criterion. From basic data $r_c = 4.0$; and, $r_s = 7.0$. Substituting these values in Equation (5), $Q_1 = 87.97$ cu ft per sec per ft. The crest length as determined by Equation (5) is $B = \frac{350}{87.97} = 3.98$ ft; and, the throat area, A_t, is $3.98 \times 3 = 11.94$ sq ft.

In practical design work it is customary to "round off" dimensions to the nearest even figure greater than that determined. In this case the throat dimensions would be taken as 4 ft by 3 ft. Obviously any change in the dimensions affects the pressures accordingly. This would introduce additional computation and slight variances from the assumed basic data, which, although not objectionable in practice, would probably result in some confusion. For this reason the dimensions will be kept in decimal form as they appear from a solution of the equation. Whatever the adopted throat dimensions are, Equation (4) in the form

$$Q_1 = V_c \, r_c \log_e \frac{r_s}{r_c} \quad \text{.......................(10)}$$

can be used to determine the minimum absolute pressure that may be expected in the upper bend. By substituting the known values of Q_1, r_c, and r_s, and solving for the velocity at the crest, it is possible to compute the corresponding pressure intensity. When the head-water is at crest elevation, the negative pressure is equal to the velocity head.

The next step is to provide some form of inlet and to fix the proportions of the upper leg. As stated previously, the design of this part of the siphon tube does not lend itself to analytical solution. Good judgment, experience, consideration of hydraulic performance, and limitations of the individual problem serve to guide the engineer in the design of the upper leg. To provide an area of from two to three times the area of the throat, an inlet opening 5 ft by 6 ft is selected. This will furnish an area of 30 sq ft or slightly greater than 2.5 times the area of the throat. Fig. 2 indicates the configuration of the spillway from the inlet to the crest section.

In the design of the lower leg the problem is to keep the tube flowing full; in other words, sufficient resistance to flow must be provided so that there is no tendency toward an increase in flow with its inevitable encroachment upon the minimum permissible absolute pressure. This desired back-pressure is created partly by frictional resistances which result in the dissipation of energy in the form of heat, and partly by the resistance offered by a reduction in outlet area which causes the remainder of the excess head to be converted into kinetic energy. To all intents and purposes the siphon tube is analogous to a large pipe line, and the total head lost plus that converted into velocity must equal the total head available. To facilitate solution of the problem let it be assumed that a transition in shape is effected between Section 3 and Section 4. Section 3 is identical to the throat section whereas Section 4, although equal in area to the throat opening, is circular. Furthermore, let it be assumed that the tube is uniform in cross-section from Section 4 to the outlet. It is possible now, having the proportions fixed, and having the discharge, to

compute the losses that occur in the entire structure. The order and magnitude of these losses, as computed for the problem at hand, are recorded in Table 1.

TABLE 1.—TABULATION OF LOSSES

Section (see Fig. 2)	Losses Expressed as a Decimal Fraction of the Velocity Head in the Throat Section					Loss, in feet	Velocity, in feet per sec	h_v, in feet	Accumulated losses plus h_v	Elevation of hydraulic gradient
	Entrance	Contraction	Bends	Friction	Total					
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1	0.032	....	....	....	0.032	0.4	16.7	4.3	4.7	75.3
2	....	0.084	0.052	0.018	0.154	2.0	29.3	13.3	15.7	64.3
3	....	....	0.160	0.044	0.204	2.7	29.3	13.3	18.4	61.6
4	....	....	....	0.048	0.048	0.6	29.3	13.3	19.0	61.0
5	....	....	....	0.245	0.245	3.2	29.3	13.3	22.2	57.8
6	....	....	0.160	0.069	0.229	3.0	29.3	13.3	25.2	54.8
7	....	....	....	0.088	0.088	1.2	29.3	13.3	26.4	53.6
8	....	....	....	0.035	0.035	0.5	65.4*	66.4	80.0	0

* Velocity in the reduced outlet area.

The values shown in Column (6), Table 1, represent the total head lost between sections expressed as a percentage of the velocity head in the throat section. A summation of the quantities in Column (6), Table 1, gives the value of K_1 in Equation (9) when $A_t = 11.94$ sq ft; $H = 80.0$ ft; $K_1 = 103.5\%$ (the summation of Column (6)); and, $h_v = 13.3$ ft. Substituting these values in Equation (9), it is found that the area to which the outlet must be reduced is 5.35 sq ft, or the diameter of the outlet will be 2.61 ft. Thus with the determination of the outlet size the problem is solved. Usually the outlet piece is a special casting and can be made to furnish the exact area required. The purpose of the reducer or choke at the outlet is to avoid excessive negative pressures, which result in unsatisfactory hydraulic performance. If an area greater than that determined herein is selected, smooth action in the operation of the spillway would be jeopardized. On the other hand, if greater constriction is provided, other conditions being equal, the only adverse effect would be a reduction in the spillway capacity. This is undesirable economically since the remainder of the siphon tube is not being used to the greatest advantage.

In the computation of the head losses appearing in Columns (2), (3), (4), and (5), Table 1, the writer used standard forms accepted in practice. The actual coefficients used are not shown since it is believed that a discussion relative to the evaluation of head losses might detract from the original purpose of the paper. Other ramifications might be introduced such as moving the reducer nearer to the throat or providing a gradual taper instead of a relatively sudden contraction. The writer is confident that the problem, as solved herein, together with the simplifying assumptions introduced, serves adequately

to elucidate the fundamental principles involved. The engineer frequently finds it necessary to use the standard pipe available in stock sizes. This may be done, but caution must always be used and the final layout must be investigated to be sure that the revised proportions do not induce pressures below the minimum permissible absolute pressure.

Columns (7) to (11), Table 1, are self-explanatory and were computed so that a plot of the hydraulic gradient could be drawn. Since Column (6) represents the total head lost between sections, expressed as a decimal fraction of the velocity head in the throat section, Column (7) is obtained simply by multiplying the values in Column (6) by 13.3 ft, or the velocity head in the throat section. The drop in the hydraulic gradient from the level of the headwater surface at any section is equal to the sum of the accumulated losses plus the velocity head at that section. Using the center line of the outlet as datum, Column (11) shows the elevation of the hydraulic gradient at any section. A graphical plot of the gradient is shown in Fig. 3 and illustrates quite clearly its limitations as well as its application. The condition of flow in the upper and lower bends is such as to invalidate the pressures shown by the computed hydraulic gradient in those regions. For the sake of illustration the siphon tube is distorted by plotting length along the spillway axis as the abscissa against elevation as the ordinate. With such an arrangement it is possible to measure pressures at any point along a vertical line. The preceding text has demonstrated that a computed hydraulic gradient could not be utilized to predict localized pressure intensities at the crest. In Fig. 3 it would seem that the negative pressures to expect at the crest and the crown at the summit would be 15.7 ft and 18.7 ft of water, respectively. Actual prototype measurements and model experiments prove conclusively that such deduction is erroneous. The order of magnitude of pressure variations in the upper bend, as computed by assuming free vortex flow, gives results closely in accordance with actual flow conditions. In the basic data for the problem under consideration the outlet was assumed to be discharging into atmosphere; hence the hydraulic gradient must intersect the siphon tube at its outlet. Fig. 3 indicates how the formula derived and the procedure outlined so adjusts the proportions of the spillway that for the design discharge of 350 cu ft per sec the head losses sustained through the entire structure, plus the exit velocity head, bring the gradient down to the proper elevation at the outlet.

In this paper the writer has intentionally made little or no reference to details such as priming, air vents, ejectors, sealing basins, etc. (A. H. Naylor in 1935[2] and the late G. F. Stickney, M. Am. Soc. C. E., in 1922,[4] have offered excellent discussions of these features.) This was done, not because they are unimportant, but because the phase of siphon design dealt with is complete without them. Naturally each high-head siphon spillway that is designed presents individual problems. The writer does not wish to create the impression that the simple application of the specialized formulas constitutes an "iron-clad," fool-proof method of solution. It is necessary to have a clear conception of the laws governing siphonic action as well as an understanding knowledge of basic hydraulic principles involved.

[4] *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1098.

Conclusion

In the design of a high-head siphon spillway for which it is desired to fix the proportions so that a definite capacity can be provided, there are two considerations of paramount importance: (1) The provision of adequate cross-sectional area at the throat, thereby preventing the occurrence of excessive negative pressures; and (2) the provision of a reduced outlet area to prevent excessive pressures and consequent intermittent and highly unsatisfactory operation.

The first requirement can be met by assuming free vortex flow in the upper bend. Such an assumption very closely approximates actual flow conditions in that part of the siphon tube. A study of the hydraulic gradient will not reveal true localized pressure variations at the summit.

To determine the proper degree of constriction at the outlet without encroaching upon the full capacity of the throat opening it is necessary to analyze the head losses in the siphon and so adjust the outlet area that the hydraulic gradient intersects the lower leg at its extremity. If the siphon does not discharge into atmosphere the gradient must indicate the pressure at the outlet.

Acknowledgment

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PAPERS

STRESS DISTRIBUTION AROUND A TUNNEL

By Raymond D. Mindlin,[1] Assoc. M. Am. Soc. C. E.

Synopsis

The problem of the disturbance of stress around a tunnel is treated in this paper from the point of view of the theory of elasticity. The tunnel is considered to be a horizontal cylindrical hole of circular cross-section situated in a semi-infinite elastic solid under the action of gravity. Through the introduction of the bipolar co-ordinate system an exact solution of the classical elasticity equations is obtained which satisfies the boundary conditions at both the upper free surface of the solid and the free surface of the hole.

Formulas for the stresses are given in terms of infinite series. The series have been evaluated and the results are given in the form of tables and curves which show that the stresses are important when the hole is either very close to the upper surface of the solid or very far from the surface.

Introduction

In the study of the equilibrium of elastic solids, problems are often encountered in which the shape of the structure and the distribution of the loading are so irregular that the ordinary methods of mechanics and strength of materials are not adequate to cope with the situation. The analysis of the disturbance of stress around a tunnel presents just these difficulties. This problem is concerned with an elastic solid of large dimensions, pierced by a hole (the tunnel) and bounded by an unloaded upper surface (Fig. 1). Before this mass is pierced by the tunnel, it is in a condition of elastic equilibrium. The problem considered here is that of the disturbance of this original condition of stress created when the mass is pierced by the tunnel.

The methods of the theory of elasticity have been adapted previously to studies of the tunnel problem.[2] The most nearly complete discussions were given by S. Yamaguti,[3] and H. Schmidt,[4] who investigated the state of stress

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[2] Since this paper was submitted Prof. Z. Anzo has called the author's attention to a paper "On the Stresses in a Gravitating Elastic Body Having a Circular Hole near Its Horizontal Surface," by Z. Anzo (in Japanese), Technology Reports of the Kyushu Imperial University, Vol. XII, No. 3, June, 1937.

[3] "On the Stresses Around a Horizontal Circular Hole in a Gravitating Elastic Solid," by S. Yamaguti, *Journal* of the Civil Engineering Society of Japan, Vol. 15 (1929), pp. 291–303.

[4] "Statische Probleme des Tunnel- und Druckstollenbaues," by H. Schmidt, 1926.

around a horizontal hole of circular cross-section, situated in a gravitating solid of indefinite extent. In his analysis, Yamaguti correctly considered the effect on the stresses of (1) the weight of the material removed from the hole, and (2) the stress concentration induced by an assumed type of initial stress in the surrounding medium. However, the influence of conditions on the upper surface of the solid was neglected. Accordingly, by St. Venant's Principle, the solution is applicable only when the distance of the hole from the upper surface is large in comparison with the radius of the hole. When such a restriction is made, however, a complicated solution of the type used by Yamaguti is no longer necessary because, in this case, the effect of the weight of the material removed from the hole is negligible in comparison with the stress concentration induced by the initial stress in the surrounding material. When the hole is far from any other boundary the latter effect can be obtained quickly by a method[5] well known in the theory of elasticity. The hole, with its center at point A (Fig. 1), is considered simply as situated in a uniform stress field composed of the principal stresses which were initially present at point A.

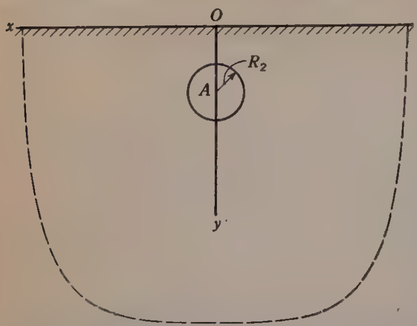


FIG. 1.—HORIZONTAL CLYINDRICAL HOLE OF CIRCULAR CROSS-SECTION IN A GRAVITATING, SEMI-INFINITE, ELASTIC SOLID

An exact solution of the elasticity equations is described here in which full consideration is given to the conditions at both the boundary of the hole and the free upper surface of the solid. The results are valid, therefore, regardless of the proximity of the two boundaries. In fact, the two boundaries may be taken so close together that the solution may be applied to the study of the dead load stresses in the thin-arched structure included between them. A simple method for treating this case is given in Appendix I. Finally, more detailed consideration is given to the assumed state of initial stress in the solid, which need not be the particular state assumed by Yamaguti.

NOTATION

The symbols used in this paper are defined where they first appear and are assembled alphabetically, for reference, in Appendix II.

STATEMENT OF THE PROBLEM

The tunnel is considered to be a horizontal cylindrical cavity of circular cross-section with its axis parallel to the z-axis of a rectangular co-ordinate system x, y, z. The surrounding material has, as its upper surface, the plane $y = 0$, but is otherwise unbounded. Technically such a body is known as a semi-infinite solid. The y-axis penetrates, in its positive direction, vertically downward and bisects the cross-section of the tunnel (see Fig. 1). The semi-infinite solid has a uniform weight per unit of volume, w, so that the problem involves a constant body force, w, per unit of volume, acting in the positive y direction.

[5] "Theory of Elasticity," by S. Timoshenko, New York, 1934, p. 80.

The length of the tunnel is assumed to be large in comparison with its diameter. This factor and the uniformity of the body force permit a treatment of the problem as one of plane strain.[6]

One method of studying problems in plain strain involves expressing the components of stress in terms of a body-force potential, Ω, and derivatives of a stress function χ. In rectangular co-ordinates these expressions take the form[7]

$$\sigma_x = \frac{\partial^2 \chi}{\partial y^2} + \Omega \quad \text{(1a)}$$

$$\sigma_y = \frac{\partial^2 \chi}{\partial x^2} + \Omega \quad \text{(1b)}$$

and,

$$\tau_{xy} = -\frac{\partial^2 \chi}{\partial x \, \partial y} \quad \text{(1c)}$$

The components (X, Y) of the body force per unit of volume are derived from the body-force potential by

$$X = -\frac{\partial \Omega}{\partial x} \quad \text{(2a)}; \qquad \text{and,} \qquad Y = -\frac{\partial \Omega}{\partial y} \quad \text{(2b)}$$

The stress function χ is a function of x and y only and is governed by the fourth-order differential equation

$$\nabla^4 \chi = -\frac{1 - 2\nu}{1 - \nu} \nabla^2 \Omega \quad \text{(3)}$$

in which ν is Poisson's ratio and $\nabla^4 = \nabla^2 \nabla^2$ where ∇^2 is Laplace's operator $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)$.

In the present case, $X = 0$ and $Y = w$ so that, from Equations (2), $\Omega = -w\,y$; whence $\nabla^2 \Omega = 0$ and Equation (3) becomes

$$\nabla^4 \chi = 0 \quad \text{(4)}$$

The problem, then, is that of finding a solution of Equation (4) that satisfies a particular set of boundary conditions. The straight boundary and the circular boundary, in this case, are unloaded, and, therefore, there will be four boundary conditions to the effect that the normal stress and the shearing stress on each of these boundaries is zero.

On a horizontal plane, at a great distance below the tunnel, there will be zero shearing stress and a uniform normal stress equal to $-w\,y$. On vertical planes normal to the x-axis and situated at a great distance from the tunnel, the distribution of stress will depend upon the state of stress assumed to have existed in the semi-infinite solid before the boring of the tunnel. Three initial states will be assumed.

Case I.—It may be assumed, for example, that each element of the solid is initially under a state of isotropic pressure. The stress in the earth at great

[6] "A Treatise on Photo-Elasticity," by E. G. Coker and L. N. G. Filon, Cambridge, University Press, 1931, p. 125.

[7] *Loc. cit.*, p. 128.

depths is probably in this state, approximately. Under this assumption the lateral boundary condition will be a linear distribution of normal pressure $\sigma_x = - w\, y$ as shown in Fig. 2(a). Such a stress distribution in the semi-

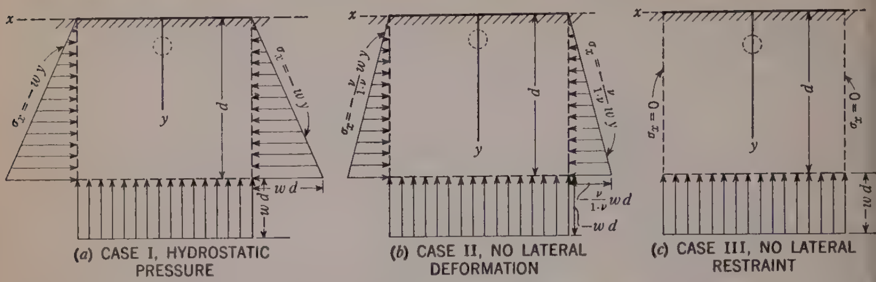


FIG. 2.—ASSUMED BOUNDARY CONDITIONS FOR THE SEMI-INFINITE SOLID

infinite solid, without the hole, may be expressed by means of the body-force potential

$$\Omega = - w\, y \qquad (5a)$$

and the stress function

$$\chi_1 = 0 \qquad (5b)$$

Case II.—It might be assumed, however, that initially (long before the boring of the tunnel) the material was restrained from lateral displacement during a supposed application of the gravitational field. In this case it may be shown that the lateral pressure is

$$\sigma_x = - \frac{\nu}{1 - \nu} w\, y$$

and that the initial state of stress may be expressed by

$$\Omega = - w\, y \qquad (6a)$$

and

$$\chi_2 = \frac{1 - 2\,\nu}{6(1 - \nu)} w\, y^3 \qquad (6b)$$

This is the state of initial stress (shown in Fig. 2(*b*)) which was assumed by Yamaguti.

Case III.—Another assumption which might be made is that there is no lateral stress at all (Fig. 2(*c*)) in which case the initial stress system may be represented by

$$\Omega = - w\, y \qquad (7a)$$

and

$$\chi_3 = \tfrac{1}{6} w\, y^3 \qquad (7b)$$

These three assumptions include a great range of possibilities for the initial state of stress.

CURVILINEAR CO-ORDINATES

Returning to a consideration of the four boundary conditions on the straight and circular boundaries, it is known that the mathematical form will be greatly

simplified if the analysis is framed in a system of curvilinear co-ordinates which is particularly suited to the boundaries. The curvilinear co-ordinate system (α, β) which will be used is obtained through a conformal transformation of the type[8]

$$x + i\,y = f(\alpha + i\,\beta) \qquad (8)$$

in which $i = \sqrt{-1}$. Important parts will be taken in the solution by the stretch ratio, J, of the transformation and the angle ϕ which the tangent to the curve $\beta = constant$ makes with the x-axis. It is known[9] that these functions are given by

$$J\,e^{i\phi} = f'(\alpha + i\,\beta) \qquad (9)$$

in which e = base of Naperian logarithms. The expressions for the radii of curvature of the co-ordinate curves will be used also. If ρ_1 and ρ_2 are the radii of curvature of $\beta = constant$ and $\alpha = constant$, respectively (both positive for counterclockwise rotation of the tangent moving in the positive direction along the curve), then[9]

$$\frac{1}{\rho_1} = \frac{1}{J}\frac{\partial \phi}{\partial \alpha} = -\frac{1}{J^2}\frac{\partial J}{\partial \beta} \qquad (10a)$$

and

$$\frac{1}{\rho_2} = \frac{1}{J}\frac{\partial \phi}{\partial \beta} = \frac{1}{J^2}\frac{\partial J}{\partial \alpha} \qquad (10b)$$

Finally, the components of stress in curvilinear co-ordinates are given by[10]

$$\sigma_\alpha = \frac{1}{J^2}\frac{\partial^2 \chi}{\partial \beta^2} + \frac{1}{J^3}\left(\frac{\partial \chi}{\partial \alpha}\frac{\partial J}{\partial \alpha} - \frac{\partial \chi}{\partial \beta}\frac{\partial J}{\partial \beta}\right) + \Omega \qquad (11a)$$

$$\sigma_\beta = \frac{1}{J^2}\frac{\partial^2 \chi}{\partial \alpha^2} + \frac{1}{J^3}\left(\frac{\partial \chi}{\partial \beta}\frac{\partial J}{\partial \beta} - \frac{\partial \chi}{\partial \alpha}\frac{\partial J}{\partial \alpha}\right) + \Omega \qquad (11b)$$

and

$$\tau_{\alpha\beta} = -\frac{1}{J^2}\frac{\partial^2 \chi}{\partial \alpha\,\partial \beta} + \frac{1}{J^3}\left(\frac{\partial \chi}{\partial \alpha}\frac{\partial J}{\partial \beta} + \frac{\partial \chi}{\partial \beta}\frac{\partial J}{\partial \alpha}\right) \qquad (11c)$$

Bipolar Co-ordinates

A co-ordinate system that is ideally suited to the present requirements is the bipolar system. It was first applied to two-dimensional elasticity by G. B. Jeffery,[11] who gave a general solution of the elasticity equations in this co-ordinate system for the case of zero body force. His method will be adapted to the present requirements after account has been taken of the effect of the action of the gravitational force.

To obtain a system of bipolar co-ordinates, Equation (8) is particularized as

$$x + i\,y = i\,a \coth \tfrac{1}{2}(\alpha + i\,\beta) \qquad (12)$$

[8] "A Treatise on Photo-Elasticity," by E. G. Coker and L. N. G. Filon, Cambridge, University Press, 1931, p. 152.

[9] *Loc. cit.*, p. 153.

[10] *Loc. cit.*, p. 163.

[11] "Plane Stress and Plane Strain in Bipolar Coordinates," by G. B. Jeffery, *Transactions* of the Royal Society, London, England, Series A, Vol. 221 (1920), pp. 265–293.

in which a = distance from the origin to a pole, in bipolar co-ordinates. Equating real and imaginary parts in Equation (12) there results

$$x = \frac{a \sin \beta}{\cosh \alpha - \cos \beta} \quad \text{(13a)}$$

and

$$y = \frac{a \sinh \alpha}{\cosh \alpha - \cos \beta} \quad \text{(13b)}$$

From Equations (9) and (12)

$$J\, e^{i\phi} = -\frac{i\,a}{2} \operatorname{csch}^2 \tfrac{1}{2}(\alpha + i\,\beta) \quad \text{(14)}$$

whence

$$J = \frac{a}{\cosh \alpha - \cos \beta} \quad \text{(15a)}$$

and,

$$\tan \phi = \frac{\cosh \alpha \cos \beta - 1}{\sinh \alpha \sin \beta} \quad \text{(15b)}$$

Equation (12) may also be written in the form

$$\alpha + i\,\beta = \log \frac{x + i(y + a)}{x + i(y - a)} \quad \text{(16)}$$

Writing $x + i(y + a) = r_1\, e^{i\theta_1}$ and $x + i(y - a) = r_2\, e^{i\theta_2}$, and equating real and imaginary parts in Equation (16), there results

$$\alpha = \log \frac{r_1}{r_2} \quad \text{(17a)}$$

and

$$\beta = \theta_1 - \theta_2 \quad \text{(17b)}$$

in which r_1 and r_2 are the radii to a point in the x, y-plane from the points $(0, -a)$, $(0, +a)$, respectively, and θ_1, θ_2 are the angles which these radii make with the x-axis, as shown in Fig. 3.

The curves α = *constant* are coaxial circles of radius $a \operatorname{csch} \alpha$ having the poles $(0, -a)$ and $(0, +a)$ as limiting points. The centers of these circles lie on the y-axis at distances $a \coth \alpha$ from the origin. The x-axis is the curve $\alpha = 0$. This and a circle α = *constant* (positive) represent the boundaries of the problem.

The curves β = *constant* are circular arcs passing through the poles $(0, \pm a)$; β is the angle included between the radii r_1 and r_2; it has a discontinuity of 2π on passing between the poles.

In terms of θ_1 and θ_2, the angle ϕ which the tangent to β = *constant* makes with the x-axis is given by

$$\phi = \theta_1 + \theta_2 + \frac{\pi}{2} \quad \text{(18)}$$

From Equations (11) and (15a) the components of stress in bipolar co-or-

dinates are found to be

$$a\sigma_\alpha = \left\{(\cosh\alpha - \cos\beta)\frac{\partial^2}{\partial\beta^2} - \sinh\alpha\frac{\partial}{\partial\alpha} - \sin\beta\frac{\partial}{\partial\beta} + \cosh\alpha\right\}\left(\frac{\chi}{J}\right) + a\,\Omega \;..(19a)$$

$$a\sigma_\beta = \left\{(\cosh\alpha - \cos\beta)\frac{\partial^2}{\partial\alpha^2} - \sinh\alpha\frac{\partial}{\partial\alpha} - \sin\beta\frac{\partial}{\partial\beta} + \cos\beta\right\}\left(\frac{\chi}{J}\right) + a\,\Omega \;..(19b)$$

and

$$a\tau_{\alpha\beta} = -\,(\cosh\alpha - \cos\beta)\frac{\partial^2}{\partial\alpha\,\partial\beta}\left(\frac{\chi}{J}\right) \;\ldots\ldots\ldots\ldots (19c)$$

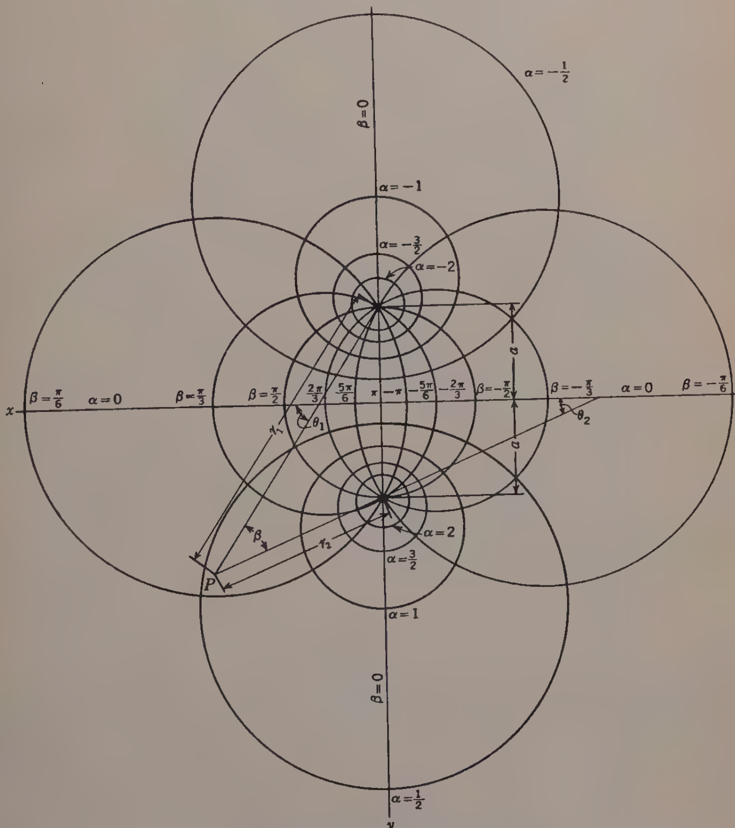


FIG. 3.—BIPOLAR CO-ORDINATES

BOUNDARY CONDITIONS

In each of the three cases, there are two curves, α = *constant*, which are free of stress. Since $\tau_{\alpha\beta} = 0$ on a free boundary α = *constant*, Equation (19c) requires that

$$\frac{\partial}{\partial\alpha}\left(\frac{\chi}{J}\right) = \xi \text{ (a constant)} \;\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

On the same boundary $\sigma_\alpha = 0$. Hence, noting that $-\,w\,y = -\,w\,J\sinh\alpha$: and substituting Equation (20) in Equation (19a) with $\sigma_\alpha = 0$, there results,

$$\left\{(\cosh\alpha - \cos\beta)\frac{\partial^2}{\partial\beta^2} - \sin\beta\frac{\partial}{\partial\beta} + \cosh\alpha\right\}\left(\frac{\chi}{J}\right) = w\,a\,J\sinh\alpha + \xi\sinh\alpha \quad (21)$$

Equation (21) is an ordinary differential equation the solution of which is found by the usual methods[12] to be

$$\frac{\chi}{J} = \xi\tanh\alpha + \eta\,(\cosh\alpha\cos\beta - 1) + \zeta\sin\beta + w\,a^2\operatorname{csch}\alpha - \tfrac{1}{2}\,w\,a^2\operatorname{csch}^2\alpha\,\phi\sin\beta \quad (22)$$

in which ξ, η, and ζ are constants.

Equations (20) and (22) are the conditions which $\frac{\chi}{J}$ must satisfy on each boundary α = *constant*.

The last term in Equation (22) warrants special notice. In the stress function

$$\chi = -\frac{w\,a^2}{2}\operatorname{csch}^2\alpha_1\,\phi\,J\sin\beta \quad (23)$$

it is observed that $a \operatorname{csch}\alpha_1$ is the radius, R_2, of the circle $\alpha = \alpha_1$; and that $\phi = \theta_1 + \theta_2 + \frac{\pi}{2}$ and $J\sin\beta = x$. Hence Equation (23) may be written as

$$\chi = -\tfrac{1}{2}\,w\,R_2^2\,x\left(\theta_1 + \theta_2 + \frac{\pi}{2}\right) \quad (24)$$

Equation (24) is recognized as the stress function for a pair of single forces[13] acting at the poles in the direction of the negative y-axis. The magnitudes of the forces are equal to $w\,\pi\,R_2^2$ which is the weight per unit of thickness of the material included in the circle $\alpha = \alpha_1$. Hence Equation (23) is the stress function required to annul the resultant of the stresses produced on α = *constant* by the body force.

Early Terms of the Stress Function

In order to satisfy the boundary conditions on a boundary α = *constant*, the following procedure suggested by Jeffery[11] will be followed. The resultant of whatever stresses exist on the boundary will be annulled first and then a general method, due to Jeffery, will be applied to eliminate the remaining stresses.

It was seen in the preceding section that the resultant of the stresses on a boundary $\alpha = \alpha_1$ may be removed by the stress function given in Equation (23). This function introduces many valued terms[14] in the displacements. In the present problem it is required that the displacements be single-valued in the region $\alpha > 0$. This may be accomplished by adding the stress function

$$\frac{\chi}{J} = \frac{1 - 2\nu}{4\,(1 - \nu)}\,a^2\operatorname{csch}^2\alpha_1\,[\alpha\sinh\alpha - \alpha\,(\cosh\alpha - \cos\beta)] \quad (25)$$

[12] "Differential Equations," by H. T. H. Piaggio, London (1929), Chapter VII.

[13] "A Treatise on Photo-Elasticity," by E. G. Coker and L. N. G. Filon, Cambridge, University Press, 1931, p. 327.

[14] "A Treatise on Photo-Elasticity," by E. G. Coker and L. N. G. Filon, Cambridge, University Press, 1931, p. 326.

Equation (25) is perhaps more readily recognized when it is written in terms of $r_1, r_2, \theta_1, \theta_2$:

$$\chi = \frac{1 - 2\nu}{4(1 - \nu)} w R_2^2 [(y - a) \log r_2 - (y - a) \log r_1] \ldots\ldots (26)$$

The term $(y - a) \log r_2$ in Equation (26) serves to eliminate the many-valued displacements in the region $\alpha > 0$ which were introduced by the term $x\,\theta_2$ in Equation (24).[13] Accordingly there should be included, in the stress function, the early terms

$$\frac{\chi_4}{J} = -\tfrac{1}{2} w a^2 \operatorname{csch}^2 \alpha_1$$

$$\times \left\{ \phi \sin \beta - \frac{1 - 2\nu}{2(1 - \nu)} [\alpha \sinh \alpha - \alpha (\cosh \alpha - \cos \beta)] \right\} \ldots\ldots (27)$$

First Auxiliary Stress Function

The stress function χ_4 leaves only a self equilibrating system of stress on $\alpha = \alpha_1$; but, at the same time, it introduces stresses on the initially free boundary $\alpha = 0$. It is advantageous to remove the stresses introduced on $\alpha = 0$ by Equation (27) before proceeding with the application of the boundary conditions, Equations (20) and (22). In this way advantage may be taken of a great simplification of the boundary equations suggested by Jeffery.

The stress function required to annul the stresses produced on $\alpha = 0$ by Equation (27) is well known.[15] In terms of the bipolar notation it is:

$$\frac{\chi_5}{J} = \frac{w a^2}{8(1 - \nu)} \operatorname{csch}^2 \alpha_1 (\cosh 2\alpha - \sinh 2\alpha - 1) \cos \beta \ldots\ldots (28)$$

It should be observed that Equations (27) and (28), combined, represent the two-dimensional solution for a force of magnitude $w \pi R_2^2$ applied along the y-axis at Point $(0, a)$ in the interior of the semi-infinite solid $y > 0$.

Second Auxiliary Stress Function

It is now required to find a second auxiliary stress function, satisfying Equation (4), which will annul the stresses on $\alpha = \alpha_1$ introduced by $\chi_{1,2,3}$, χ_4, χ_5 and Ω, and at the same time produce no stresses on $\alpha = 0$. In curvilinear co-ordinates Equation (4) becomes[16]

$$\left(\frac{\partial^2}{\partial \alpha^2} + \frac{\partial^2}{\partial \beta^2}\right)\left\{\frac{1}{J^2}\left(\frac{\partial^2 \chi}{\partial \alpha^2} + \frac{\partial^2 \chi}{\partial \beta^2}\right)\right\} = 0 \ldots\ldots (29a)$$

Jeffery showed that, for the case of bipolar co-ordinates, Equation (29a) can be reduced to a linear equation with constant coefficients by considering $\frac{\chi}{J}$

[15] "Der Spannungszustand der durch eine Einzelkraft im Innern beanspruchten Halbscheibe," by E. Melan, *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 12 (1932), pp. 343–346; see also "Stress Systems in a Circular Disk under Radial Forces," by R. D. Mindlin, *Transactions*, A. S. M. E., Vol. 59 (1937), pp. A-115 to A-118; and "Force at a Point in the Interior of a Semi-Infinite Solid," by R. D. Mindlin, *Physics*, Vol. 7 (1936), pp. 195–202.

[16] "A Treatise on Photo-Elasticity," by E. G. Coker and L. N. G. Filon, Cambridge, University Press, 1931, p. 164.

instead of χ as the dependent variable. Equation (29a) then becomes

$$\left(\frac{\partial^4}{\partial\alpha^4} + 2\frac{\partial^4}{\partial\alpha^2\,\partial\beta^2} + \frac{\partial^4}{\partial\beta^4} - 2\frac{\partial^2}{\partial\alpha^2} + 2\frac{\partial^2}{\partial\beta^2} + 1\right)\left(\frac{\chi}{J}\right) \doteq 0 \quad \ldots\ldots(29b)$$

On account of the symmetry of the present problem, the solution must be even in β. Furthermore, since there is a discontinuity of 2π in β on passing between the poles, the solution must be periodic in β, of period 2π. A solution of the form

$$\frac{\chi_6}{J} = f(\alpha)\cos n\beta \ldots\ldots\ldots\ldots(30)$$

will therefore be considered. Substituting Equation (30) in Equation (29b) there results:

$$\left(\frac{d^4}{d\alpha^4} - 2(n^2-1)\frac{d^2}{d\alpha^2} + n^4 - 2n^2 + 1\right) f(\alpha) = 0 \ldots\ldots(31)$$

Equation (31) is an ordinary differential equation whose solution is found to be, by the usual methods:[17]

For $n \geqq 2$,

$$f_n(\alpha) = A_n\cosh(n+1)\alpha + B_n\cosh(n-1)\alpha + C_n\sinh(n+1)\alpha + D_n\sinh(n-1)\alpha \ldots\ldots(32a)$$

for $n = 1$,

$$f_1(\alpha) = A_1\cosh 2\alpha + B_1 + C_1\sinh 2\alpha + D_1\alpha \ldots\ldots(32b)$$

and, for $n = 0$,

$$f_0(\alpha) = A_0\cosh\alpha + B_0\alpha\cosh\alpha + C_0\sinh\alpha + D_0\alpha\sinh\alpha \ldots\ldots(32c)$$

Now, $A_0\cosh\alpha$ and $B_1\cos\beta$ give the same stresses; hence, A_0 may be taken equal to zero, which is equivalent to adding a constant term to the stress function. An inspection of Equations (20) and (22), with $w = 0$, reveals that χ_6 will produce no stresses on $\alpha = 0$, if

$$f(0) = 0 \ldots\ldots\ldots\ldots(33a)$$

and

$$f'(0) = 0 \ldots\ldots\ldots\ldots(33b)$$

Substituting Equations (32) in Equations (33), the following relations are obtained among the constants, which will assure the vanishing of σ_α and $\tau_{\alpha\beta}$ when $\alpha = 0$:

For $n \geqq 1$,

$$A_n + B_n = 0 \ldots\ldots\ldots\ldots(34a)$$

for $n \geqq 2$,

$$(n+1)C_n + (n-1)D_n = 0 \ldots\ldots\ldots\ldots(34b)$$

Also

$$2C_1 + D_1 = 0 \ldots\ldots\ldots\ldots(34c)$$

and,

$$B_0 + C_0 = 0 \ldots\ldots\ldots\ldots(34d)$$

Jeffery has shown that the condition that the displacements be single valued requires that

$$D_0 = 0 \ldots\ldots\ldots\ldots(35a)$$

[17] "Differential Equations," by H. T. H. Piaggio, London (1929), Chapter III.

and

$$D_1 + B_0 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (35b)$$

Substituting Equations (34) and (35) in Equations (32) and writing E_n for $\dfrac{C_n}{(n-1)}$, the following form is found for the second auxiliary stress function:

$$\frac{\chi_6}{J} = B_0 \left[\alpha (\cosh \alpha - \cos \beta) + \sinh \alpha (\cosh \alpha \cos \beta - 1)\right]$$
$$+ A_1 (\cosh 2\alpha - 1) \cos \beta + \sum_{n=2}^{\infty} \{A_n [\cosh (n+1)\alpha - \cosh (n-1)\alpha]$$
$$+ E_n [(n-1) \sinh (n+1)\alpha - (n+1) \sinh (n-1)\alpha]\} \cos n\beta \ldots (36)$$

(The line of reasoning leading from Equations (33) to (36) is essentially that followed by Jeffery[11] in applying his general bipolar solution to the problem of a semi-infinite plate containing an unstressed circular hole, and under a uniform tension parallel to its straight edge. The process has been repeated here because Jeffery considered correctly only the first of the conditions given in Equations (34), with the result that the term $B_0 \sinh \alpha (\cosh \alpha \cos \beta - 1)$ is lacking in his Equation (66).[11] Consequently, in his application, the boundary conditions on $\alpha = 0$ are not satisfied.)

Complete Stress Function

The complete stress function (χ) is the sum of the initial stress function (χ_1, χ_2, or χ_3), the early terms (χ_4) and the two auxiliary stress functions (χ_5 and χ_6), as follows:

$$\frac{\chi}{J} = \frac{\chi_{1,\,2,\,3}}{J} - \frac{w\, a^2}{2} \operatorname{csch}^2 \alpha_1$$
$$\times \left\{ \phi \sin \beta - \frac{1 - 2\nu}{2(1-\nu)} [\alpha \sinh \alpha - \alpha (\cosh \alpha - \cos \beta)] \right\}$$
$$+ \frac{1}{8(1-\nu)} w\, a^2 \operatorname{csch}^2 \alpha_1 (\cosh 2\alpha - \sinh 2\alpha - 1) \cos \beta$$
$$+ B_0 [\alpha (\cosh \alpha - \cos \beta) + \sinh \alpha (\cosh \alpha \cos \beta - 1)]$$
$$+ A_1 (\cosh 2\alpha - 1) \cos \beta + \sum_{n=2}^{\infty} \Big\{ A_n [\cosh (n+1)\alpha - \cosh (n-1)\alpha]$$
$$+ E_n [(n-1) \sinh (n+1)\alpha - (n+1) \sinh (n-1)\alpha] \Big\} \cos n\beta \ldots (37)$$

There remains only the determination of the arbitrary constants. This is accomplished by substituting Equation (37) in Equations (20) and (22) with the proper value for $\chi_{1,\,2,\,3}$ for each of Cases I, II and III.

Case I

In this case the first term on the right in Equation (37) is $\dfrac{\chi_1}{J} = 0$. The determination of the constants B_0, A_1, A_n, E_n by substitution of Equation (37) in the boundary conditions (Equations (20) and (22)) is perfectly straightforward except for one consideration. The difficulty arises from the appearance of a term $\dfrac{\sin^2 \beta}{(\cosh \alpha_1 - \cos \beta)}$ which results from the differentiation of the function $\phi \sin \beta$ with respect to α in Equation (20).

The values of the constants are determined by solving a set of simultaneous linear equations formed by equating coefficients of like terms of the type $\cos n\beta$. The term involving $(\cosh\alpha_1 - \cos\beta)$ in the denominator can only be adapted to this process by expanding it in infinite series as follows:

$$\frac{\sin^2\beta}{\cosh\alpha_1 - \cos\beta} = e^{-\alpha_1} + e^{-2\alpha_1} - 2\sinh\alpha_1 \sum_{n=2}^{\infty} e^{-n\alpha_1}\cos n\beta \quad \ldots\ldots (38)$$

This is valid for $\alpha_1 > 0$. It is the appearance of this term in the first boundary condition which necessitates an infinite series solution. Were it not for this term the constants A_n and E_n would vanish and the final stress function would appear as the sum of a finite number of elementary functions.

Using Equation (38) after the substitution of Equation (37) in Equations (20) and (22), the following values for the constants result:

$$B_0 = -\tfrac{1}{2} w\, a^2 \operatorname{csch}^2\alpha_1 \left[2\coth\alpha_1 + \frac{1-2\nu}{2(1-\nu)}(\coth\alpha_1 - 1)\right] \ldots (39a)$$

$$A_1 = \tfrac{1}{2} w\, a^2 \operatorname{csch}^2\alpha_1 \ldots\ldots (39b)$$

$$A_n = -\tfrac{1}{2} w\, a^2 \operatorname{csch}^2\alpha_1 \times \frac{e^{-n\alpha_1}\sinh\alpha_1(\sinh n\alpha_1\cosh\alpha_1 - n\cosh n\alpha_1\sinh\alpha_1)}{\sinh^2 n\alpha_1 - n^2\sinh^2\alpha_1} \ldots\ldots (39c)$$

and

$$E_n = -\tfrac{1}{2} w\, a^2 \operatorname{csch}^2\alpha_1 \frac{e^{-n\alpha_1}\sinh^2\alpha_1\sinh n\alpha_1}{\sinh^2 n\alpha_1 - n^2\sinh^2\alpha_1} \ldots\ldots (39d)$$

Substituting Equations (39) in Equation (37), the stress function for Case I is found to be:

$$\begin{aligned}\frac{\chi}{J} = -\tfrac{1}{2} w\, a^2 \operatorname{csch}^2\alpha_1 \Bigg\{ &\phi\sin\beta - \frac{1-2\nu}{2(1-\nu)}\alpha\sinh\alpha \\ &+ \frac{5-6\nu}{2(1-\nu)}\coth\alpha_1(\cosh\alpha - \cos\beta)\alpha - \frac{3-4\nu}{4(1-\nu)}(\cosh 2\alpha - 1)\cos\beta \\ &+ \frac{1}{2}\left(\frac{5-6\nu}{2(1-\nu)}\coth\alpha_1 - 1\right)\sinh 2\alpha\cos\beta + \sum_{n=2}^{\infty}\frac{2\sinh\alpha_1\, e^{-n\alpha_1}}{\sinh^2 n\alpha_1 - n^2\sinh^2\alpha_1} \\ &\times[\sinh n\alpha_1 \sinh n\alpha \sinh(\alpha - \alpha_1) \\ &- n\sinh\alpha_1\sinh\alpha\sinh n(\alpha - \alpha_1)]\Bigg\}\cos n\beta \quad \ldots\ldots (40)\end{aligned}$$

The most important stresses are those in the circular boundary. Using Equations (19b) and (40):

$$\begin{aligned}[\sigma_\beta]_{\alpha=\alpha_1} = \frac{w\, a(\cosh\alpha_1 - \cos\beta)}{2\sinh^2\alpha_1}\Bigg\{ &\frac{2(1 - \cosh\alpha_1\cos\beta)\sinh\alpha_1}{(\cosh\alpha_1 - \cos\beta)^2} \\ &- 4\cosh\alpha_1 - \left(\frac{5-6\nu}{1-\nu} + 2e^{-2\alpha_1}\right)\cos\beta \\ &- 4\sinh\alpha_1\sum_{n=2}^{\infty} N_n\cos n\beta\Bigg\} \quad \ldots\ldots (41)\end{aligned}$$

in which

$$N_n = \frac{n\, e^{-n\alpha_1}\,(\sinh n\,\alpha_1 \cosh n\,\alpha_1 - n \sinh \alpha_1 \cosh \alpha_1)}{\sinh^2 n\,\alpha_1 - n^2 \sinh^2 \alpha_1} \quad \ldots\ldots (42)$$

In order to obtain numerical values for $[\sigma_\beta]_{\alpha=\alpha_1}$ it is necessary to tabulate values for the series in Equation (41); but it is found that the series converges very slowly, especially for small values of α_1. To facilitate the computations, the more slowly converging part may be separated by letting

$$N_n = n\, e^{-n\alpha_1} + R_n \quad \ldots\ldots (43)$$

Noting that

$$2 \sum_{n=1}^{\infty} n\, e^{-n\alpha_1} \cos n\,\beta = \frac{\cosh \alpha_1 \cos \beta - 1}{(\cosh \alpha_1 - \cos \beta)^2} \quad \ldots\ldots (44)$$

and substituting Equation (43) in Equation (41):

$$[\sigma_\beta]_{\alpha=\alpha_1} = \frac{2\,w\,a\,(\cosh \alpha_1 - \cos \beta)}{\sinh \alpha_1} \left\{ \frac{1 - \cosh \alpha_1 \cos \beta}{(\cosh \alpha_1 - \cos \beta)^2} - \coth \alpha_1 - \frac{(7 - 8\,\nu)}{4\,(1 - \nu)} \frac{\cos \beta}{\sinh \alpha_1} + 2\, e^{-\alpha_1} \cos \beta - \sum_{n=2}^{\infty} R_n \cos n\,\beta \right\} \quad \ldots\ldots (45)$$

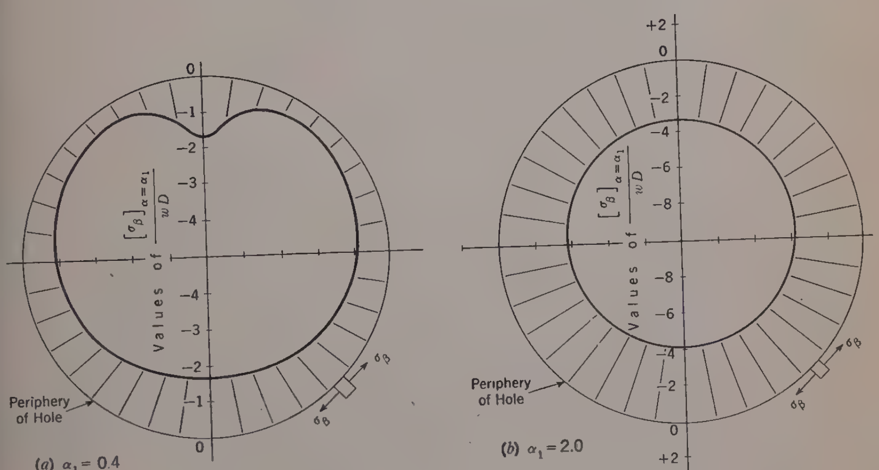


FIG. 4.—DIAGRAM OF STRESSES $\frac{[\sigma_\beta]_{\alpha=\alpha_1}}{w\,D}$ AROUND THE TUNNEL

Values of R_n are given in Table 1. Values of $[\sigma_\beta]_{\alpha=\alpha_1}$ around the hole have been calculated for a wide range of values of α_1 and for three values (0, $\frac{1}{4}$, and $\frac{1}{2}$) of ν (see Table 2). In the typical curves in Fig. 4 the dimensionless ratio $\frac{[\sigma_\beta]_{\alpha=\alpha_1}}{w\,D}$ (in which D is the diameter of the hole) is plotted radially from the periphery of the hole. Fig. 4(*a*) gives the variation of $[\sigma_\beta]_{\alpha=\alpha_1}$ around the hole for a small value (0.4) of α_1—that is, when the hole is close to the surface; and Fig. 4(*b*) is for a larger value (2.0) of α_1. Both are for $\nu = \frac{1}{2}$.

TABLE 1.—VALUES OF R_n

R_n	α_1										
	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4
R_2	1.4663	0.5193	0.2043	0.0848	0.0364	0.0157	0.0069	0.0031	0.0014	0.0006	0.0003
R_3	0.7620	0.1852	0.0482	0.0127	0.0033	0.0034	0.0002	0.0001	....	....	....
R_4	0.3848	0.0615	0.0099	0.0016	0.0003	....	....	....	....	....	....
R_5	0.1884	0.0189	0.0018	0.002	....	....	....	....	....	....	....
R_6	0.0893	0.0054	0.0003	....	....	....	....	....	....	....	....
R_7	0.0409	0.0015	....	....	....	....	....	....	....	....	....
R_8	0.0181	0.0004	....	....	....	....	....	....	....	....	....
R_9	0.0078	0.0001	....	....	....	....	....	....	....	....	....
R_{10}	0.0033	....	....	....	....	....	....	....	....	....	....
R_{11}	0.0013	....	....	....	....	....	....	....	....	....	....
R_{12}	0.0005	....	....	....	....	....	....	....	....	....	....
R_{13}	0.0002	....	....	....	....	....	....	....	....	....	....
R_{14}	0.0001	....	....	....	....	....	....	....	....	....	....

Table 3 gives some idea of the relation between the radius (R_2) of the hole and the distance (c) of the center of the hole from the free surface for the range of α_1 for which calculations were made.

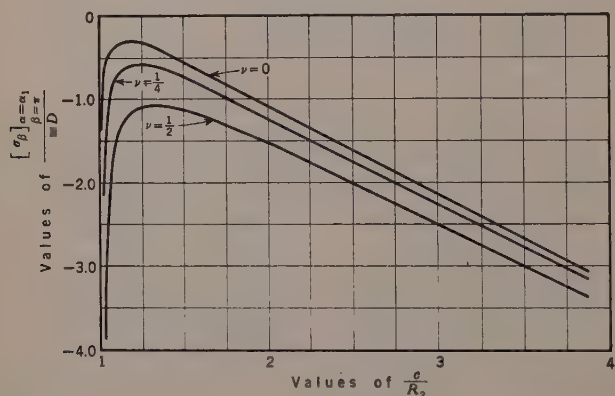


FIG. 5.—VARIATION OF THE QUANTITY $\frac{[\sigma_\beta]_{\alpha=\alpha_1,\ \beta=\pi}}{w\,D}$ AT THE HIGHEST POINT OF THE TUNNEL WALL *vs.* THE RATIO $\frac{c}{R_2}$ FOR CASE I

In Fig. 5 the quantity $\frac{[\sigma_\beta]_{\alpha=\alpha_1}}{w\,D}$ at the point of the hole nearest to the upper free surface is plotted against the ratio $\frac{c}{R_2}$ for three values of Poisson's ratio.

Figs. 4 and 5 and Table 2 illustrate the following significant facts for Case I:

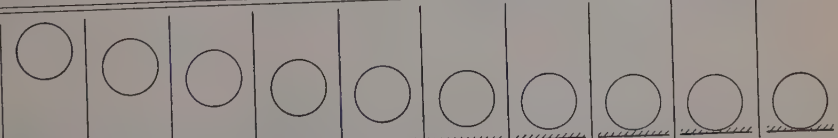
(1) The disturbing influence of the upper free boundary is not felt until $\frac{c}{R_2}$ is less than about 1.5;

(2) For $\frac{c}{R_2} > 1.5$, the stresses increase almost linearly with the depth;

TABLE 2.—VALUES OF $\dfrac{[\sigma_\beta]_{\alpha=\alpha_1}}{w\,D}$ (CASE I)

α_1	β = 0	20	40	60	80	100	120	140	160	180
				$\nu = 0$ (CASE I)						
0.2	−1.63	....	....	....	....	....	....	....	....	−1.35
0.4	−1.69	−1.07	−0.77	−0.71	−0.69	−0.65	−0.57	−0.49	−0.43	−0.40
0.6	−1.80	−1.42	−1.13	−0.93	−0.86	−0.76	−0.62	−0.46	−0.34	−0.29
0.8	−1.96	−1.72	−1.37	−1.20	−1.07	−0.93	−0.74	−0.59	−0.45	−0.40
1.0	−2.18	−2.01	−1.71	−1.49	−1.33	−1.17	−0.98	−0.80	−0.66	−0.61
1.2	−2.46	−2.33	−2.07	−1.84	−1.64	−1.46	−1.27	−1.08	−0.95	−0.90
1.4	−2.81	−2.71	−2.48	−2.24	−2.03	−1.82	−1.64	−1.45	−1.31	−1.26
1.6	−3.25	−3.17	−2.97	−2.73	−2.50	−2.28	−2.07	−1.89	−1.76	−1.71
1.8	−3.79	−3.72	−3.54	−3.31	−3.07	−2.84	−2.62	−2.44	−2.31	−2.27
2.0	−4.45	−4.42	−4.22	−4.00	−3.75	−3.51	−3.29	−3.11	−2.98	−2.94
				$\nu = \frac{1}{4}$ (CASE I)						
0.2	−1.63	....	....	....	....	....	....	....	....	−2.18
0.4	−1.67	−1.05	−0.73	−0.66	−0.66	−0.69	−0.74	−0.78	−0.81	−0.85
0.6	−1.77	−1.39	−1.02	−0.89	−0.84	−0.79	−0.73	−0.66	−0.60	−0.58
0.8	−1.93	−1.68	−1.33	−1.16	−1.05	−0.96	−0.83	−0.74	−0.65	−0.62
1.0	−2.14	−1.97	−1.67	−1.45	−1.31	−1.19	−1.06	−0.93	−0.83	−0.79
1.2	−2.41	−2.29	−2.03	−1.80	−1.63	−1.48	−1.34	−1.19	−1.09	−1.06
1.4	−2.76	−2.66	−2.44	−2.21	−2.01	−1.84	−1.69	−1.55	−1.43	−1.39
1.6	−3.19	−3.11	−2.92	−2.69	−2.49	−2.30	−2.13	−1.98	−1.88	−1.84
1.8	−3.73	−3.66	−3.49	−3.27	−3.04	−2.85	−2.67	−2.52	−2.42	−2.38
2.0	−4.39	−4.36	−4.17	−3.96	−3.74	−3.53	−3.34	−3.19	−3.08	−3.05
				$\nu = \frac{1}{2}$ (CASES I AND II)						
0.2	−1.61	....	....	....	....	....	....	....	....	−3.85
0.4	−1.64	−0.99	−0.63	−0.54	−0.59	−0.78	−1.06	−1.35	−1.58	−1.67
0.6	−1.73	−1.33	−0.93	−0.80	−0.79	−0.85	−0.95	−1.05	−1.12	−1.15
0.8	−1.86	−1.61	−1.25	−1.08	−1.01	−1.01	−1.01	−1.04	−1.05	−1.06
1.0	−2.06	−1.89	−1.58	−1.37	−1.28	−1.23	−1.20	−1.18	−1.16	−1.15
1.2	−2.32	−2.20	−1.94	−1.73	−1.60	−1.52	−1.46	−1.41	−1.38	−1.37
1.4	−2.66	−2.56	−2.35	−2.13	−1.98	−1.87	−1.81	−1.74	−1.69	−1.67
1.6	−3.08	−3.00	−2.82	−2.62	−2.46	−2.33	−2.24	−2.16	−2.11	−2.09
1.8	−3.61	−3.55	−3.39	−3.20	−3.02	−2.88	−2.77	−2.69	−2.63	−2.61
2.0	−4.26	−4.24	−4.06	−3.88	−3.71	−3.56	−3.44	−3.35	−3.29	−3.27

TABLE 3.—POSITION OF HOLE WITH RESPECT TO HORIZONTAL SURFACE FOR VALUES OF α_1 AND $\dfrac{c}{R_2}$



α_1	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
$\dfrac{c}{R_2}$	1.02	1.08	1.19	1.34	1.54	1.81	2.15	2.58	3.10	3.76

(3) For $\frac{c}{R_2} < 1.2$, the stresses increase with great rapidity as the hole approaches the upper surface of the solid;

(4) For materials of reasonable strength and density, the stresses are important only when the hole is very close to the upper surface of the solid or when it is very far from the surface;

(5) Variation of Poisson's ratio, ν, has little effect on the stresses except when the hole is very close to the upper surface; and,

(6) The stresses on the boundary of the tunnel are greatest at the lowest point of the tunnel cross-section except when α_1 is very small, in which case there are two equal maxima straddling the highest point.

In an analysis of this type, involving so much mathematical detail, it is important to determine whether or not the solution approaches, in a limiting case, a result that can be obtained directly by a simple process. When the hole is far from the upper surface, there are two factors that should be revealed in the solution: First, the stresses induced by the weight of the material removed from the hole should be very small in comparison with the stress concentration due to the initial stress in the surrounding material; and, second, by St. Venant's principle, the latter effect should be the same as for a hole in a uniform-stress field composed of the principal stresses present before the introduction of the hole. In Case I, the initial stress system is an isotropic pressure, and it is known[5] that in such a system the introduction of a hole doubles the stress.

To perform the limiting process in Equation (41), let $\alpha \longrightarrow \infty$ while $a \longrightarrow c$; $a \operatorname{csch} \alpha_1 \longrightarrow R_2$; and $\beta \longrightarrow \psi$ in which $\psi \left(= \frac{3\pi}{2} - \phi \right)$ is the angle between the radius from the center of the hole and the normal to the straight boundary, so that

$$\cos \psi = \frac{\cosh \alpha_1 \cos \beta - 1}{\cosh \alpha_1 - \cos \beta} \qquad \text{(46a)}$$

and

$$\sin \psi = \frac{\sinh \alpha_1 \sin \beta}{\cosh \alpha_1 - \cos \beta} \qquad \text{(46b)}$$

Making these substitutions in Equation (41):

$$[\sigma_\beta]_{\alpha=\alpha_1} = - 2\, c\, w - R_2\, w \frac{3 - 4\,\nu}{2\,(1 - \nu)} \cos \psi \qquad \text{(47)}$$

In Equation (47) the second term arises from the weight of the material removed from the hole and the first term is the stress concentration effect. The initial stress at a distance c below the surface is $-\, c\, w$, so that the term $-\, 2\, c\, w$ reveals the predicted stress concentration factor of 2. If R_2 is small in comparison with c, the second term is small in comparison with the first. Therefore, if the hole is far from the upper surface, it is only necessary to invoke St. Venant's principle to arrive immediately at the resulting stresses.

Cases II and III

The same general stress function, Equation (37), which was used for Case I is applicable to Cases II and III. The latter two differ from the former only

in the first term of Equation (37). In Case I the first term was obtained from Equation (5b). In Case II, the first term is obtained from Equation (6b); and for Case III, Equation (7b) is used. Thus Cases II and III differ from Case I only by a term

$$\chi_{2,\,3} = G\,w\,y^3 \qquad \text{(48)}$$

in which, for Case II,

$$G = \frac{1 - 2\,\nu}{6\,(1 - \nu)} \qquad \text{(49a)}$$

and, for Case III,

$$G = \tfrac{1}{6} \qquad \text{(49b)}$$

To arrive at Cases II and III, therefore, it is only necessary to superpose on Case I the stress system arising from the initial stress represented by Equation (48). This stress system is obtained by combining the second auxiliary stress function (Equation (36)) with Equation (48) in such a manner that the stresses on the circle $\alpha = \alpha_1$ are annulled. Equation (48) can only be used with Equation (36) if it is expressed in the form $J\,f(\alpha)\cos n\,\beta$. To this end write

$$\chi_{2,\,3} = G\,w\,y^3 = \frac{G\,w\,a^3 \sinh^3 \alpha}{(\cosh \alpha - \cos \beta)^3}$$

$$= G\,w\,a^2\,J\left\{\cosh \alpha + 2\sum_{n=1}^{\infty} (\cosh \alpha + n \sinh \alpha)\,e^{-n\alpha} \cos n\,\beta\right\} \qquad \text{(50)}$$

The complete stress function to be added to Case I to transform it to Cases II and III is, therefore,

$$\frac{\chi}{J} = G\,w\,a^2 \Biggl\{\cosh \alpha + 2\sum_{n=1}^{\infty} (\cosh \alpha + n \sinh \alpha)\,e^{-n\alpha} \cos n\,\beta$$

$$+ B_0\,[\alpha\,(\cosh \alpha - \cos \beta) + \sinh \alpha\,(\cosh \alpha \cos \beta - 1)]$$

$$+ A_1\,(\cosh 2\,\alpha - 1)\cos \beta + \sum_{n=2}^{\infty} \Bigl(A_n\,[\cosh (n + 1)\,\alpha - \cosh (n - 1)\,\alpha]$$

$$+ E_n\,[(n - 1)\sinh (n + 1)\,\alpha - (n + 1)\sinh (n - 1)\,\alpha]\Bigr) \cos n\,\beta\Biggr\} \qquad \text{(51)}$$

To determine the values of the constants, Equation (51) is substituted in Equations (20) and (22) with $w = 0$ in the latter; thus:

$$B_0 = 3 \coth \alpha_1 \operatorname{csch}^2 \alpha_1 \qquad \text{(52a)}$$

$$A_1 = -\tfrac{3}{2} \operatorname{csch}^2 \alpha_1 \qquad \text{(52b)}$$

$$A_n = -\frac{n\,(n^2 - 1)\sinh^2 \alpha_1}{\sinh^2 n\,\alpha_1 - n^2 \sinh^2 \alpha_1} \qquad \text{(52c)}$$

and

$$E_n = \frac{n \sinh \alpha_1\,(n \sinh \alpha_1 + \cosh \alpha_1) + e^{-n\alpha_1} \sinh n\,\alpha_1}{\sinh^2 n\,\alpha_1 - n^2 \sinh^2 \alpha_1} \qquad \text{(52d)}$$

Again, the stresses $[\sigma_\beta]_{\alpha=\alpha_1}$ on the boundary of the hole are found by substituting Equations (52) and (51) in Equation (19b). The result is

$$[\sigma_\beta]_{\alpha=\alpha_1} = w\,G\,a\,(\cosh \alpha_1 - \cos \beta)\{6 \coth \alpha_1 \operatorname{csch} \alpha_1$$

$$+ 6 \operatorname{csch}^2 \alpha_1 \cos \beta + 4 \sinh \alpha_1 \sum_{n=2}^{\infty} S_n \cos n\,\beta\} \qquad \text{(53)}$$

in which

$$S_n = \frac{n\,(n^2 - 1)\sinh n\,\alpha_1}{\sinh^2 n\,\alpha_1 - n^2 \sinh^2 \alpha_1} \qquad (54)$$

The series in Equation (53) converges too slowly for calculation purposes. The slowly converging part may be separated out by letting

$$S_n = 2\,n\,(n^2 - 1)\,e^{-n\alpha_1} + T_n \qquad (55)$$

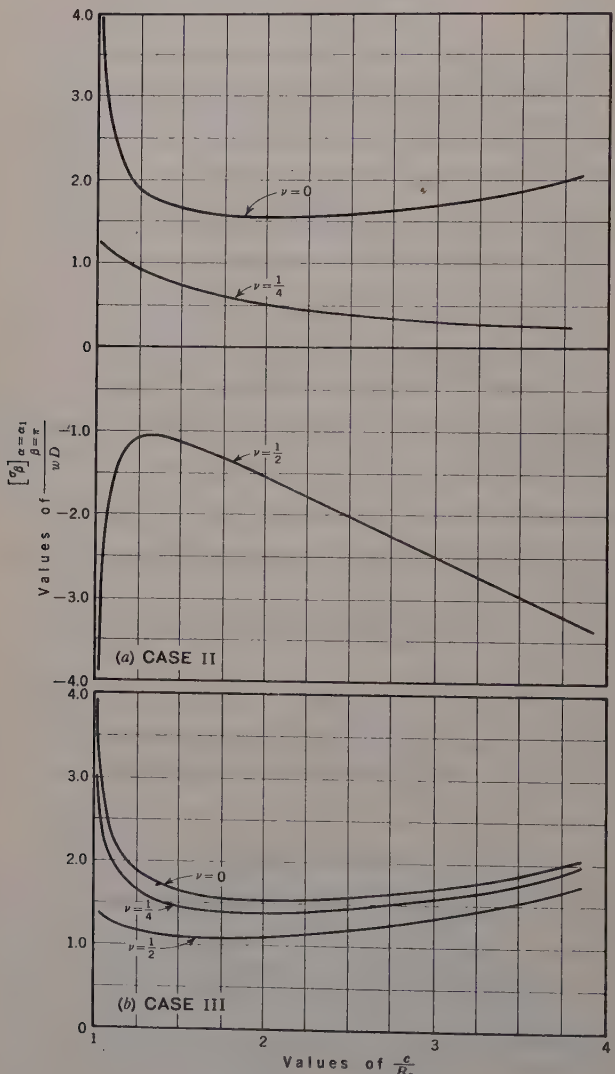


FIG. 6.—VARIATION OF THE QUANTITY $\frac{[\sigma_\beta]_{\alpha=\alpha_1, \beta=\pi}}{w\,D}$ AT THE HIGHEST POINT OF THE TUNNEL WALL *vs.* THE RATIO $\frac{c}{R_2}$

Then, noting that

$$8\,(\cosh\alpha_1 - \cos\beta)\sum_{n=1}^{\infty} n\,(n^2-1)\,e^{-n\alpha_1}\cos n\,\beta = 6\,\mathrm{csch}^2\,\alpha_1$$
$$\times\,(2\cosh\alpha_1\cos 2\,\psi + \cos\psi + \cos 3\,\psi)\ldots\ldots\ldots\ldots(56)$$

and substituting Equation (55) in Equation (53):

$$[\sigma_\beta]_{\alpha=\alpha_1} = w\,G\,a\,(\cosh\alpha_1 - \cos\beta)\,\{6\coth\alpha_1\,\mathrm{csch}\,\alpha_1 + 6\,\mathrm{csch}^2\,\alpha_1\cos\beta$$
$$+\,4\sinh\alpha_1\sum_{n=2}^{\infty} T_n\cos n\,\beta\} + 6\,w\,G\,a\,\mathrm{csch}\,\alpha_1$$
$$\times(\cos\psi + 2\cosh\alpha_1\cos 2\,\psi + \cos 3\,\psi)\ldots\ldots\ldots\ldots(57)$$

Values of T_n are given in Table 4. Values of $[\sigma_\beta]_{\alpha=\alpha_1}$ for Case II and Case III are obtained by adding Equations (45) and (57) with G given by Equations (49a) and (49b), respectively. The results are given in Table 5.

The curves in Fig. 6 show the variation of the stress in terms of $\frac{[\sigma_\beta]_{\alpha=\alpha_1}}{w\,D}$, at the highest point of the tunnel as $\frac{c}{R_2}$ is varied. The conclusions that were drawn for Case I also hold for Cases II and III with the following additional comments:

(1) The wide variation of the stresses for the three separate cases shows the necessity of knowing the initial state of stress in the solid; and,

(2) The apparent great change in the stresses of Case II with variation of Poisson's ratio is to be attributed to variation of the state of initial stress rather than to any marked change with Poisson's ratio.

TABLE 4.—VALUES OF T_n

T_n	α_1										
	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4
T_2	41.4132	10.1670	3.3052	1.2281	0.4924	0.2047	0.0876	0.0382	0.0168	0.0074	0.0013
T_3	33.2080	6.1642	1.4073	0.3146	0.0867	0.0219	0.0055	0.0026	0.0003	0.0001	....
T_4	20.1894	3.1329	0.4583	0.0680	0.0099	0.0015	0.0001	....	....	....	....
T_5	16.2229	1.3750	0.1211	0.0103	0.0008	0.0001	....	....	....	....	....
T_6	10.1179	0.5326	0.0271	0.0012	0.0001	....	....	....	....	....	....
T_7	5.9144	0.1857	0.0053	0.0001	....	....	....	....	....	....	....
T_8	3.2620	0.0593	0.0005	....	....	....	....	....	....	....	....
T_9	1.7069	0.0176	0.0001	....	....	....	....	....	....	....	....
T_{10}	0.8523	0.0043	....	....	....	....	....	....	....	....	....
T_{11}	0.4100	0.0023	....	....	....	....	....	....	....	....	....
T_{12}	0.1882	0.0002	....	....	....	....	....	....	....	....	....
T_{13}	0.0828	....	....	....	....	....	....	....	....	....	....
T_{14}	0.0361	....	....	....	....	....	....	....	....	....	....
T_{15}	0.0140	....	....	....	....	....	....	....	....	....	....
T_{16}	0.0029	....	....	....	....	....	....	....	....	....	....
T_{17}	0.0005	....	....	....	....	....	....	....	....	....	....
T_{18}	0.0001	....	....	....	....	....	....	....	....	....	....

Once again it is desirable to check the results by applying a limiting process as in the preceding section. In this case there is available, for comparison, Yamaguti's approximate solution[3] for a hole far from the upper free surface with initial stresses corresponding to Case II.

TABLE 5.—VALUES OF $\frac{[\sigma_\beta]_{\alpha=\alpha_1}}{w\,D}$ FOR CASES II AND III

α_1	β									
	0	20	40	60	80	100	120	140	160	180
	(*a*) CASE II AND CASE III, $\nu = 0$									
0.2	1.24	....	....	....	....	....	....	....	....	3.87
0.4	1.20	−1.62	−1.04	−1.03	−1.04	−0.53	0.73	1.89	2.16	2.05
0.6	1.25	−1.60	−1.69	−1.27	−1.00	−0.45	0.31	1.14	1.79	2.03
0.8	1.29	−2.43	−2.18	−1.68	−1.20	−0.63	0.15	0.93	1.54	1.77
1.0	1.36	−0.81	−2.53	−2.23	−1.47	−0.84	0.05	0.77	1.41	1.66
1.2	1.47	−0.48	−2.74	−2.89	−2.09	−1.07	−0.20	0.68	1.31	1.55
1.4	1.61	−0.22	−2.86	−3.53	−2.53	−1.44	−0.44	0.58	1.29	1.55
1.6	1.79	0.01	−2.96	−4.19	−3.59	−2.22	−0.75	0.48	1.31	1.60
1.8	2.04	0.24	−3.09	−4.89	−4.55	−3.00	−1.19	0.38	1.38	1.74
2.0	2.33	0.43	−3.24	−5.65	−5.65	−4.00	−1.79	0.19	1.52	1.98
	(*b*) CASE II, $\nu = \frac{1}{4}$									
0.2	0.29	....	....	....	....	....	....	....	....	1.29
0.4	0.26	−1.41	−0.90	−0.87	−0.89	−0.61	0.14	0.81	0.91	0.79
0.6	0.26	−1.51	−1.39	−1.11	−0.93	−0.58	−0.11	0.41	0.82	0.97
0.8	0.24	−2.15	−1.87	−1.48	−1.14	−0.75	−0.23	0.27	0.68	0.83
1.0	0.22	−1.17	−2.22	−1.95	−1.41	−0.97	−0.44	0.12	0.56	0.72
1.2	0.21	−1.05	−2.47	−2.50	−1.93	−1.22	−0.62	−0.02	0.42	0.58
1.4	0.19	−1.00	−2.69	−3.07	−2.35	−1.59	−0.89	−0.20	0.30	0.48
1.6	0.17	−0.99	−2.92	−3.67	−3.22	−2.26	−1.25	−0.40	0.17	0.37
1.8	0.15	−1.02	−3.19	−4.33	−4.03	−2.96	−1.72	−0.64	0.04	0.29
2.0	0.13	−1.12	−3.51	−5.06	−5.00	−3.85	−2.34	−0.99	−0.08	0.23
	(*c*) CASE III, $\nu = \frac{1}{4}$									
0.2	1.25	....	....	....	....	....	....	....	....	3.03
0.4	1.22	−1.60	−0.99	−0.97	−1.00	−0.57	0.57	1.61	1.78	1.61
0.6	1.27	−1.57	−1.57	−1.22	−0.97	−0.48	0.20	0.95	1.52	1.74
0.8	1.33	−2.39	−2.14	−1.64	−1.18	−0.65	0.07	0.78	1.34	1.55
1.0	1.40	−0.77	−2.49	−2.20	−1.45	−0.87	−0.13	0.64	1.25	1.48
1.2	1.52	−0.44	−2.70	−2.85	−2.08	−1.09	−0.27	0.57	1.17	1.40
1.4	1.66	−0.17	−2.81	−3.49	−2.52	−1.46	−0.50	0.48	1.16	1.41
1.6	1.85	0.07	−2.91	−4.15	−3.58	−2.23	−0.81	0.39	1.19	1.48
1.8	2.10	0.29	−3.04	−4.85	−4.53	−3.02	−1.24	0.30	1.27	1.62
2.0	2.39	0.49	−3.19	−5.61	−5.64	−4.01	−1.84	0.11	1.42	1.87
	(*d*) CASE III, $\nu = \frac{1}{2}$									
0.2	1.27	....	....	....	....	....	....	....	....	1.36
0.4	1.25	−1.54	−0.90	−0.86	−0.94	−0.66	0.25	1.03	1.01	0.78
0.6	1.32	−1.51	−1.49	−1.13	−0.93	−0.54	−0.02	0.56	1.00	1.17
0.8	1.39	−2.32	−2.05	−1.56	−1.14	−0.70	−0.11	0.48	0.94	1.11
1.0	1.48	−0.69	−2.41	−2.12	−1.42	−0.91	−0.27	0.39	0.92	1.12
1.2	1.61	−0.35	−2.61	−2.78	−2.04	−1.13	−0.39	0.35	0.89	1.09
1.4	1.76	−0.07	−2.72	−3.42	−2.49	−1.50	−0.61	0.28	0.90	1.13
1.6	1.96	0.18	−2.82	−4.08	−3.55	−2.27	−0.92	0.21	0.96	1.23
1.8	2.22	0.41	−2.94	−4.78	−4.51	−3.05	−1.34	0.13	1.06	1.39
2.0	2.52	0.61	−3.08	−5.54	−5.61	−4.04	−1.94	−0.05	1.21	1.65

Note: Case II, $\nu = \frac{1}{2}$ and Case I, $\nu = \frac{1}{2}$ are identical.

Applying the limiting conditions:

$$[\sigma_\beta]_{\alpha=\alpha_1} = \frac{w}{1-\nu}\left\{-c - \tfrac{1}{2} R_2 \cos\psi + 2(1-2\nu)\,c\cos 2\psi + (1-2\nu)\,R_2 \cos 3\psi\right\} \qquad (58)$$

In order to compare $[\sigma_\beta]_{\alpha=\alpha_1}$ in Equation (58) with Yamaguti's σ_θ (written $\overline{\theta\theta_1}$) note that his symbol θ is given by $\theta = \pi - \psi$.

When this substitution is made, the two formulas check except for an error in sign[17] in Yamaguti's work.

APPENDIX I

Dead Load Moment and Thrust in an Arch

The results of the preceding sections can be applied to the arch-like structure formed between $\alpha = 0$ and $\alpha = \alpha_1$ for small values of α_1; but, when α_1 is small, the infinite series in the stress formulas converge very slowly. This is due to the fact that, for $\alpha_1 = 0$, the series actually diverge. Consequently, the computations are exceedingly laborious. The difficulty may be avoided if it is observed that the resultant force and couple on a section β = *constant* connecting the two boundaries may be expressed in finite terms. In what follows, this statement will be proved and the results applied to Case I as an illustrative example.

Let s_α be an arc of the curve β = *constant*, measured positively in the direction of increasing α. In traveling along the curve in the positive direction, the right-hand side is called the outside and the left-hand side the inside. Let $\bar{X}$ and $\bar{Y}$ be the forces per unit of thickness parallel to the x-axis and the y-axis which the material outside of s_α exerts on the material inside of s_α per unit of length of s_α. Then, if l and m are the direction cosines of the outward drawn normal to s_α referred to the x-axis and the y-axis:[18]

$$\bar{X} = l\,\sigma_x + m\,\tau_{xy} \quad\text{(59a)}$$

and

$$\bar{Y} = l\,\tau_{xy} + m\,\sigma_y \quad\text{(59b)}$$

Now $l = \dfrac{dy}{ds_\alpha}$; $m = -\dfrac{dx}{ds_\alpha}$ and σ_x, σ_y and τ_{xy} are given, in terms of χ and Ω, by Equations (1). Hence

$$\bar{X} = \frac{dy}{ds_\alpha}\left(\frac{\partial^2\chi}{\partial y^2} + \Omega\right) + \frac{dx}{ds_\alpha}\frac{\partial^2\chi}{\partial x\,\partial y} = \frac{d}{ds_\alpha}\left(\frac{\partial\chi}{\partial y}\right) + \Omega\frac{dy}{ds_\alpha} \quad\text{(60a)}$$

and

$$\bar{Y} = -\frac{dy}{ds_\alpha}\frac{\partial^2\chi}{\partial x\,\partial y} - \frac{dx}{ds_\alpha}\left(\frac{\partial^2\chi}{\partial x^2} + \Omega\right) = -\frac{d}{ds_\alpha}\left(\frac{\partial\chi}{\partial x}\right) - \Omega\frac{dx}{ds_\alpha} \quad\text{(60b)}.$$

The x and y components of resultant force on the entire arc between the straight and circular boundaries are found by integrating Equations (60) along s_α, from the point of intersection of s_α with $\alpha = 0$ to the point of intersection of s_α with $\alpha = \alpha_1$. Calling these total x and y components H and V, respectively:

$$H = \int_0^{\alpha_1} \bar{X}\,ds_\alpha = \left[\frac{\partial\chi}{\partial y}\right]_0^{\alpha_1} + \int_0^{\alpha_1}\Omega\,dy \quad\text{(61a)}$$

and,

$$V = \int_0^{\alpha_1} \bar{Y}\,ds_\alpha = -\left[\frac{\partial\chi}{\partial x}\right]_0^{\alpha_1} - \int_0^{\alpha_1}\Omega\,dx \quad\text{(61b)}$$

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[17] The error in Yamaguti's paper first appears in his Equations (16). The second term in the second of his Equations (16) should be preceded by a plus sign instead of a minus sign. The error is carried through the remainder of the paper so that his formula for $\overline{\theta\theta_1}$ is incorrect.

[18] "A Treatise on Photo-Elasticity," by E. G. Coker and L. N. G. Filon, Cambridge University Press, 1931, p. 507.

Now $\dfrac{\partial \chi}{\partial y} = \dfrac{\partial \alpha}{\partial y}\dfrac{\partial \chi}{\partial \alpha} + \dfrac{\partial \beta}{\partial y}\dfrac{\partial \chi}{\partial \beta}$; or,

$$\frac{\partial \chi}{\partial y} = \frac{\partial \alpha}{\partial y}\left(J\frac{\partial}{\partial \alpha} + \frac{\partial J}{\partial \alpha}\right)\left(\frac{\chi}{J}\right) + \frac{\partial \beta}{\partial y}\left(J\frac{\partial}{\partial \beta} + \frac{\partial J}{\partial \beta}\right)\left(\frac{\chi}{J}\right) \ldots\ldots (62a)$$

and, $\dfrac{\partial \chi}{\partial x} = \dfrac{\partial \alpha}{\partial x}\dfrac{\partial \chi}{\partial \alpha} + \dfrac{\partial \beta}{\partial x}\dfrac{\partial \chi}{\partial \beta}$; or,

$$\frac{\partial \chi}{\partial y} = \frac{\partial \alpha}{\partial x}\left(J\frac{\partial}{\partial \alpha} + \frac{\partial J}{\partial \alpha}\right)\left(\frac{\chi}{J}\right) + \frac{\partial \beta}{\partial x}\left(J\frac{\partial}{\partial \beta} + \frac{\partial J}{\partial \beta}\right)\left(\frac{\chi}{J}\right) \ldots\ldots (62b)$$

Therefore

$$H = \left[\frac{\partial \alpha}{\partial y}\left(J\frac{\partial}{\partial \alpha} + \frac{\partial J}{\partial \alpha}\right)\left(\frac{\chi}{J}\right) + \frac{\partial \beta}{\partial y}\left(J\frac{\partial}{\partial \beta} + \frac{\partial J}{\partial \beta}\right)\left(\frac{\chi}{J}\right) + \int \Omega\, dy\right]_0^{\alpha_1} \ldots (63a)$$

and,

$$V = -\left[\frac{\partial \alpha}{\partial x}\left(J\frac{\partial}{\partial \alpha} + \frac{\partial J}{\partial \alpha}\right)\left(\frac{\chi}{J}\right) + \frac{\partial \beta}{\partial x}\left(J\frac{\partial}{\partial \beta} + \frac{\partial J}{\partial \beta}\right)\left(\frac{\chi}{J}\right) + \int \Omega\, dx\right]_0^{\alpha_1} ..(63b)$$

To find the total moment on the section, take moments of $\tau_{\alpha\beta}$ on β = *constant* about the center of the circle β = *constant* as the moment center. Then

$$M = \rho_1 \int_0^{\alpha_1} \tau_{\alpha\beta}\, ds_\alpha = \frac{J^2}{\left(\dfrac{\partial J}{\partial \beta}\right)_{\alpha 1}} \left[\frac{\partial}{\partial \beta}\left(\frac{\chi}{J}\right)\right]_0^{\alpha_1} \ldots\ldots\ldots\ldots (64)$$

In Equations (63) and (64) $\dfrac{\chi}{J}$ occurs only in the form of its boundary values and the boundary values of its first derivatives with respect to α and β; but, by Equations (20) and (22), these quantities are given in finite terms. Hence, by using Equations (20) and (22) in Equations (63) and (64), formulas for the force and couple on a section β = *constant*, connecting the two boundaries, may be found readily in finite terms.

Applying Equations (63) and (64) to Case I:

$$H = \frac{w\, a^2}{2}\left(\frac{1 - 2\,\nu}{2(1 - \nu)}\alpha_1 \operatorname{csch}^2 \alpha_1 - \coth \alpha_1\right) \ldots\ldots\ldots\ldots (65a)$$

$$V = -\frac{w\, a^2}{2}\left\{\left(\phi_1 - \frac{\pi}{2}\right)(\csc^2 \beta - \operatorname{csch}^2 \alpha_1) + \frac{2 \coth \alpha_1 \sin \beta}{\cosh \alpha_1 - \cos \beta} - \frac{(1 - \cosh \alpha_1 \cos \beta)(\cosh \alpha_1 + \cos \beta)}{\sin \beta \sinh \alpha_1 (\cosh \alpha_1 - \cos \beta)}\right\} \ldots\ldots\ldots\ldots (65b)$$

and,

$$M = \frac{w\, a^3}{2} \operatorname{csch}^2 \alpha_1 \left\{\left(\phi_1 - \frac{\pi}{2}\right)\cot \beta - \frac{\sinh \alpha_1}{\cosh \alpha_1 - \cos \beta} + \frac{5 - 6\,\nu}{2(1 - \nu)}(\alpha_1 \coth \alpha_1 - 1) + \sinh \alpha_1\, e^{-\alpha_1}\right\} \ldots\ldots\ldots\ldots (65c)$$

In terms of the dimensions shown in Fig. 7 these become

$$H = \frac{w}{2}\left\{\frac{1 - 2\,\nu}{2\,(1 - \nu)} R_2^2 \log \frac{c + a}{R_2} - a\, c\right\} \ldots\ldots\ldots\ldots (66a)$$

$$V = -\frac{w}{2}\left\{\Theta\,(R_1^2 - R_2^2) + y_1 f + x_1 c\right\} \ldots\ldots\ldots\ldots (66b)$$

and,

$$M = \frac{w}{2} R_2^2 \left\{ \Theta f - y_1 + \frac{5 - 6\nu}{2} \left(c \log \frac{c + a}{R_2} - a \right) + a^2 \frac{(c - a)}{R_2^2} \right\} \quad (66c)$$

To determine the moment (M_0) at the crown of the arch it should be observed that

$$\lim_{\beta \to \pi} \Theta \cot \beta = - \frac{\sinh \alpha_1}{\cosh \alpha_1 + 1} = - \frac{y_0}{a} \quad (67)$$

Hence

$$M_0 = \frac{w}{2} R_2^2 \left\{ - 2 y_0 + \frac{5 - 6\nu}{2} \left(c \log \frac{c + a}{R_2} - a \right) + \frac{a^2 (c - a)}{R_2^2} \right\} \quad (68)$$

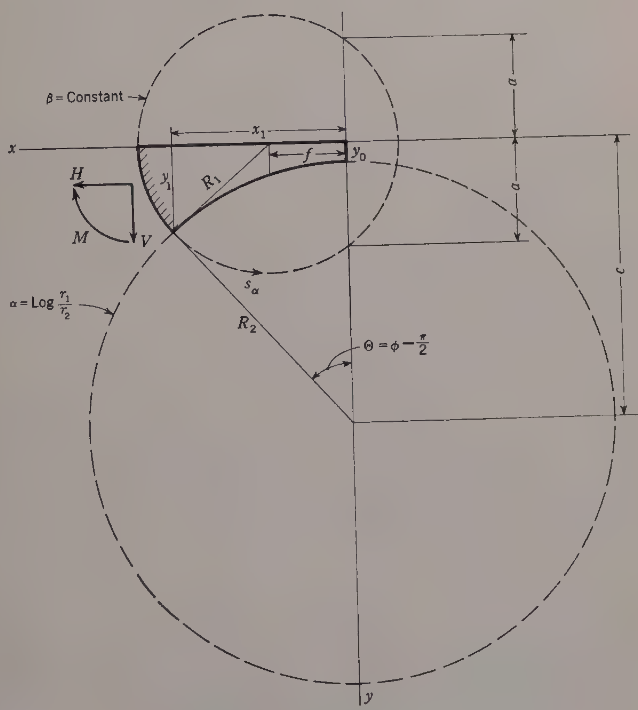

FIG. 7.—DIMENSIONS OF ARCH

APPENDIX II

NOTATION

The following notation conforms essentially with American Standard Symbols for Mechanics, Structural Engineering, and Testing Materials[19] compiled by a committee of the American Standards Association with Society representation, and approved by the Association in 1932. Where additional symbols are required, the notation used by Professor Timoshenko[20] is followed.

[19] ASA—Z10A—1932.

[20] "Theory of Elasticity," by S. Timoshenko, New York, 1934.

A = a constant ($A_0 \cdots A_n$; $B_0 \cdots B_n$; $C_0 \cdots C_n$; $D_0 \cdots D_n$);
a = a distance from the origin to a pole, in bipolar co-ordinates;
c = distance of the center of hole from the origin = $a \coth \alpha_1$
D = diameter of hole; (see also A)
d = depth below ground surface, to a horizontal plane (see Fig. 2);
e = Naperian base = 2.718 ⋯ ;
f = $a \cot \beta$ (see Fig. 7);
G = a constant defined in Equation (49);
H, V (see Fig. 7); H and V = horizontal and vertical components of a resultant force, on a section through the arch;
$i = \sqrt{-1}$;
J = stretch ratio of a conformal transformation of co-ordinates;
l, m = direction cosines (see Appendix I);
M = moment; total moment on a section through an arch; M_0 = moment at the crown of an arch (see Fig. 7);
N = a coefficient (N_n, R_n, S_n, T_n) = coefficients of Fourier series;
n = an integer (1, 2, 3 ⋯ n);
R (see N) also R_2 = radius of hole = $a \operatorname{csch} \alpha_1$; R_1 = radius of circular section through an arch = $a \csc \beta$.
r = radius; r_1 and r_2 = radii to a point from the poles of a bipolar co-ordinate system (see Fig. 3);
s = arc of a curve; s_α = element of length along a curve β = *constant;*
w = weight per unit of volume of material;
X, Y Symbols X and Y = components of body force per unit of volume; $\bar{X}$ and $\bar{Y}$ = components of stress on a unit length of curve;
x, y Symbols x and y = rectangular co-ordinates; x_1 and y_1 = rectangular co-ordinates of the point of intersection of s_α and α_1 (see Fig. 7);
α Symbols α and β = curvilinear co-ordinates; α_1 = value of the co-ordinate α on the boundary of a hole;
ζ, η, ξ = boundary constants (see Equation (22));
$\Theta = \phi - \frac{\pi}{2}$;
θ_1, θ_2 = angles that r_1 and r_2 make with the x-axis;
ν = Poisson's ratio;
ρ = radius of curvature; ρ_1 and ρ_2 = radii of curvature of the curves β = *constant* and α = *constant*, respectively;
σ = unit normal stress; σ_x, σ_y = components of unit normal stress in rectangular co-ordinates; σ_α, σ_β = components of unit normal stress in curvilinear co-ordinates;
τ = unit shear stress; τ_{xy} = unit shear stress in rectangular co-ordinates; $\tau_{\alpha\beta}$ = unit shear stress in curvilinear co-ordinates;
ϕ = angle between the x-axis and the tangent to the curve β = *constant;*
χ = stress function;
$\psi = \frac{3\pi}{2} - \phi$;
Ω = body-force function; and,
∇^2 = Laplace's two-dimensional operator ($\nabla^4 = \nabla^2\nabla^2$).

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

RECONSTRUCTION OF THE WALPOLE-BELLOWS FALLS ARCH BRIDGE

By H. E. Langley,[1] Esq., and Edward J. Ducey,[2] Esq.

Synopsis

The three-hinged, open-web, steel, arched highway bridge between the Towns of Walpole, N. H., and Bellows Falls, Vt., was one of the several crossings over the Connecticut River which were seriously damaged during the spring flood of 1936. For example, Fig. 1(*a*) shows the New Hampshire end of the span, Fig. 1(*b*), the Vermont end; and Fig. 1(*c*) is a detail view of the damage to the bottom chord. It is apparent that the lower chord sections and the adjacent web members of the up-stream truss below high-water level were so badly crippled that it was necessary to close the bridge to all vehicular traffic; and the responsible authorities were compelled to decide between major repairs to the structure and its replacement with a new one.

This bridge had been in service only since 1908, and the steelwork in general had been carefully maintained so that a considerable expenditure for repairing the damage was warranted. The consulting engineers who designed the structure[3] were called upon to submit a design for repairs. They formulated a general plan involving supports near each end, with four along the interior of the arch, all supported on timber pile groups. Detailed plans were prepared, and bids were taken for the work on this basis. The successful bid ($120 000) included all operating hazards and was somewhat in excess of the anticipated costs. The danger element involved in this undertaking was relatively high because a further accident to the main span might not only result in the loss of the structure but would probably involve difficulties at the Bellows Falls power plant, situated immediately below the bridge, which is a main link in the electric power system that serves this region. In studying the situation the Bridge Division of the New Hampshire State Highway Department decided that it was more feasible for the State to assume the risks incident to this extra-

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[2] Designing Engr., Am. Bridge Co., Pittsburgh, Pa.

[3] *Transactions*, Am. Soc. C. E., December, 1908, Vol. LXI, p. 253.

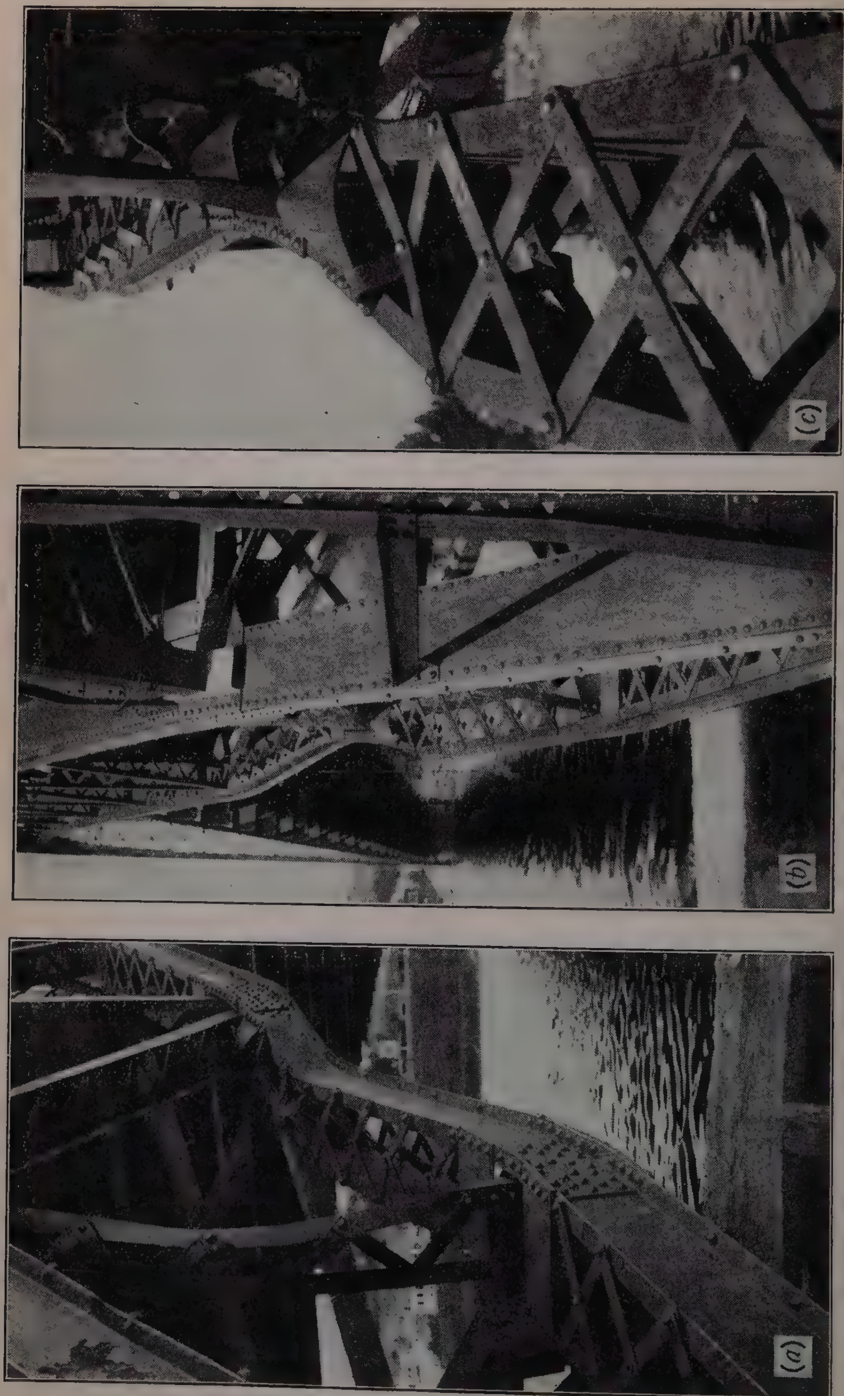


Fig. 1.—Damage Sustained by Lower Chord and Adjacent Members

hazardous procedure than the contractor. To do this it was necessary to let the fabrication and erection contract on a "cost-plus" basis to a competent structural steel contractor who would co-operate with the State forces in driving the piles and placing the concrete.

In consultation with the contractor's engineers a study was made to eliminate some of the more hazardous features of the undertaking by keeping the river as clear of falsework as the condition of the structure would admit. Accordingly, a plan was devised incorporating the use of temporary cable ties (see Fig. 3) to take the horizontal thrust of the arch, thus converting the span temporarily into a tied three-hinged arch to be carried on falsework towers near the original supports. With the falsework at the ends of the span it could not only be protected, if necessary, from floating ice by means of ice breakers or fenders extending out from the river banks, but it could be constructed from the shores as well.

The use of temporary cable ties and other features introduced in this work seem somewhat unusual, and it is believed that a description of the procedure followed will be of interest to engineers and contractors generally.

Introduction

The original structure consisted of two ribs in each truss—a top chord and a bottom chord—each being a parabola of the same general dimensions, spaced one above the other 14 ft apart vertically. The top chord consisted of two 15-in. channels with a cover plate 24 in. wide and laced at the bottom flanges. The bottom chord consisted of two similar channels laced top and bottom, with additional side plates making the gross area approximately equal to that of the top chord. The diagonal members were made up of two 5 in. by $3\frac{1}{2}$ in. by $\frac{3}{8}$ in. angles, generally, single laced. This construction is shown quite clearly in Fig. 2. Since the two ribs are parabolas of the same dimensions, the thrust line under uniform load would be midway between them because the spring-line pins and the pin at the crown were placed midway between the two. The ribs in each truss were drawn together in the last two panels at either end of the span to meet at the spring-line pins so that the thrust from the two main arch trusses was carried by the skew-back shoes to the concrete abutments which were supported principally on wooden piles. The floor system, which was hung from the trusses with loop-rod hangers, had a system of lateral or wind bracing, which had for the chords of the system two members in the planes of the two main trusses. These extended throughout the length of the span, and were connected to the bottom chords of the main trusses by slotted connections to provide expansion due to temperature changes. Although these members were comparatively light, being made up of two 6-in. by 4-in. by $\frac{1}{2}$-in. angles, together with the diagonal bracing in the top-chord lateral system, they played an important part in saving the structure from collapse when the up-stream truss was damaged.

A very interesting and complete description of the construction of this bridge, together with discussions covering the erection of this and similar structures, was presented by Lewis D. Rights, M. Am. Soc. C. E., on April, 1, 1908.[3]

FIG. 2.—VIEW OF DAMAGED MEMBERS AT THE VERMONT END OF THE SPAN

THE PROBLEM

At the time the structure was built the low-water level was several feet below the present pool elevation, but improvements in the power development facilities on the Connecticut River have raised the normal pool to its present level (Elevation 115.5) a foot or so above the original spring-line pins. The depth of water now varies from about 19 ft at the abutments to about 50 ft at mid-channel. Borings taken at the abutments and at the temporary tower sites indicated about 20 ft of mud, sand, and boulders which gradually consolidated into a series of alternate strata of hard sand and gravel at a depth of about 35 ft, underlaid with solid rock at about 60 ft to 70 ft below the river-bed. The high water during the flood of 1936, which caused the damage to the main span, reached Elevation 125.5 (see Fig. 3) and covered the roadway at the New Hampshire end of the span.

The direct damage was caused by the ice jam which collected back of the main arch and extended entirely across the river. The impact and the sustained pressure from this ice so disabled the up-stream truss that the lower chord ceased to function as originally designed, and the greater part of its share of the load was distributed to the top chord. The diagonals $U_2 L_3$ and $U_3 L_4$ and the vertical $U_3 L_3$ (Fig. 3) assumed part of the thrust from the lower chord and distributed it to the top chord, while the ties which formed the chords of the lateral bracing in the floor took up the greater part of the remainder. Although the crippled chord sections undoubtedly carried some stress, it was

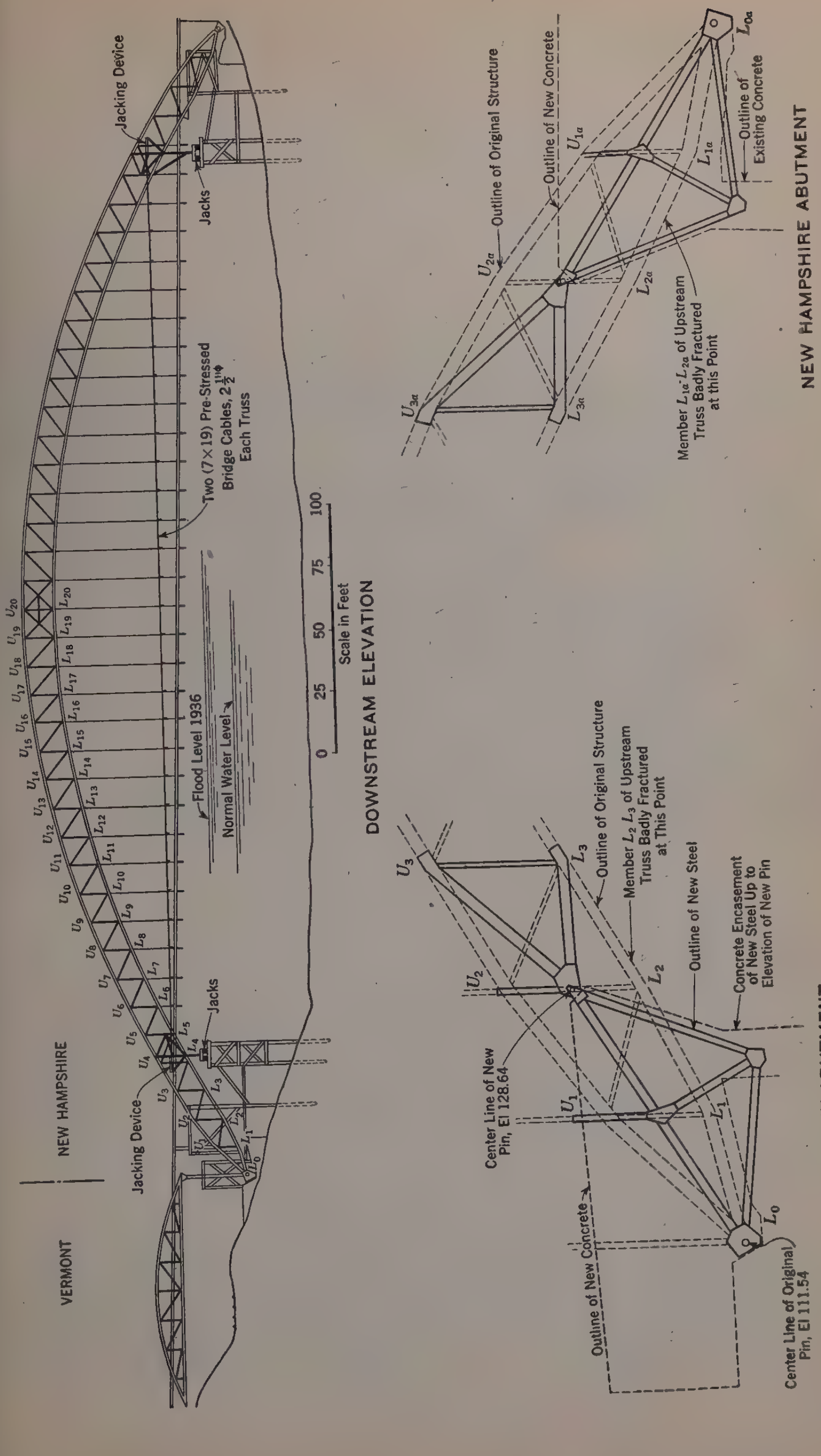


FIG. 3.—ADOPTED PLAN OF FALSEWORK FOR ALTERATIONS AND REPAIRS

largely of a secondary nature, rather than direct thrust, serving mainly to prevent the top chord from buckling upward under the thrust from the diagonals. There was a particularly noticeable tendency for the top chord to yield directly over the damaged bottom-chord sections. This was overcome effectively at the New Hampshire end by the use of ballast, the State Highway Department having placed about 50 tons of sand and gravel on the floor at this point immediately after the accident to the span. At the Vermont end 12 in. by 12 in. timber shores were wedged between the end shoe of the approach span and the haunches of the arch so as to load the haunches.

Evidently, the top chord of the up-stream truss was partly relieved of the load distributed to it from the lower chord, by the action of the top-chord lateral system which, apparently, transferred part of the load across to the down-stream truss that was not directly damaged by the ice jam. The diagonals that appeared to take the thrust from the lower rib of the up-stream truss were manifestly in serious danger of buckling under direct stress, because there was a very evident bow or lateral deflection in several of these members that disappeared after the span was picked up on the falsework, and the condition of stress in the web members throughout the structure was decidedly uncertain.

A careful survey made soon after the flood subsided revealed that the damaged up-stream truss had dropped about 6 in. at the crown and raised about 2 in. at the haunches. The crippled section of chord at the New Hampshire end shortened about $1\frac{3}{4}$ in. and the corresponding chord section at the Vermont end shortened about 3 in. Although the affected chords were battered down stream under the impact of the ice, the entire span was found to have moved up stream so that, at the temporary tower on the New Hampshire end, it was necessary to move it back down stream $2\frac{1}{2}$ in., and at the Vermont tower it was required to move it back $4\frac{3}{4}$ in. to align the structure with the original skew-backs after removing the damaged ends of the trusses. The shoes were not disturbed, of course, so that this displacement was caused by a side lurch of the structure, which was quite noticeable when viewed endwise along the bridge. The span leaned up stream, being about 1 ft out of plumb at the crown. Emergency repairs included the timber bracing, shown in Figs. 2 and 4, framed and wedged into the plane of the lower chord to prevent further side lurch.

In considering the repairs to be made to the structure it was imperative that some provision should be made to prevent a recurrence of the damage. Manifestly the lower chords of the span were quite exposed to the impact of floating ice, and it was obvious that the damaged steelwork would have to be replaced. It was observed that by removing the three panels of truss at each end, and by replacing the old members with the arrangement shown in Fig. 3, the new steel below high-water level could be encased in heavily armored reinforced concrete. This would not only protect the steel but it would also provide somewhat more lateral clearance for the floating ice at high-water level. It would thus provide a less vulnerable structure and, incidently, it would maintain the same type of structure with a 54 ft shorter span. In this arrangement the main thrust under uniform load is carried, by a single member, from the new pin set on the original

thrust line to the original skew-back. The effect of partial loading is taken care of through the cantilever action of the new steel below the pin, which steel is supported by a group of four steel piles under each truss, at a new shoe set approximately 20 ft forward of the skew-back shoes.

The obvious uncertainty of stress throughout the structure, and the general instability of the span, injected a hazard into the problem which became an important factor in the procedure to be followed in making the repairs. Little could be done toward making preliminary repairs, other than the emergency repairs already described, to secure the structure sufficiently to carry erection loads. It was impossible, therefore, to construct temporary falsework by

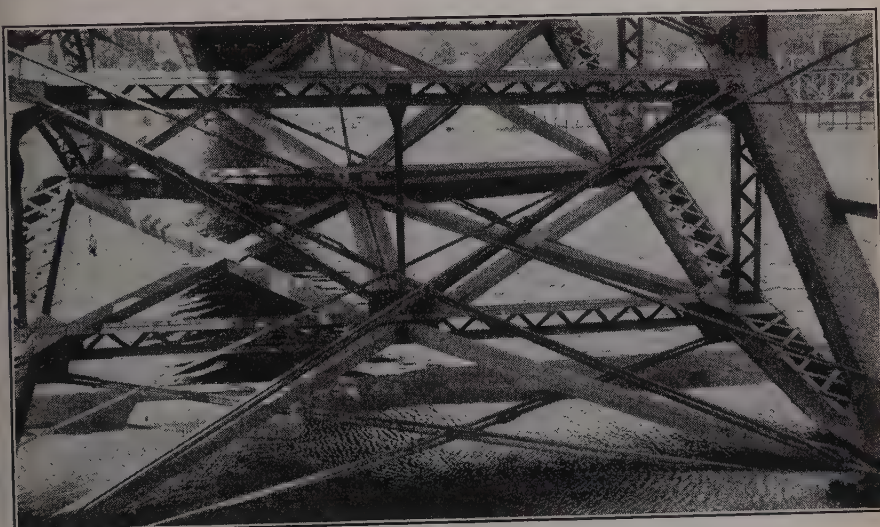


Fig. 4.—Emergency Bracing in the Plane of the Bottom Chord; View Facing Along the Bridge Toward New Hampshire

working from the span, and it was impractical to reach the site with floating equipment because of the location and the proximity of the power company's dam situated in the river immediately below the span. Therefore, in addition to keeping the river as free from falsework as possible, the plan of procedure which seemed most feasible was the one involving the least number of temporary towers and so devised that they could be erected by working out from the shores on temporary bents, keeping entirely free of the damaged span so as to disturb it as little as possible.

The Plan Adopted

The plan decided upon was to pick up the span on temporary wooden pile towers placed as close to the shores as the steel to be removed would permit, and to take the thrust by means of temporary ties, and then remove the damaged ends and replace the old steel with new material after the structure had been leveled up and re-aligned on the temporary towers.

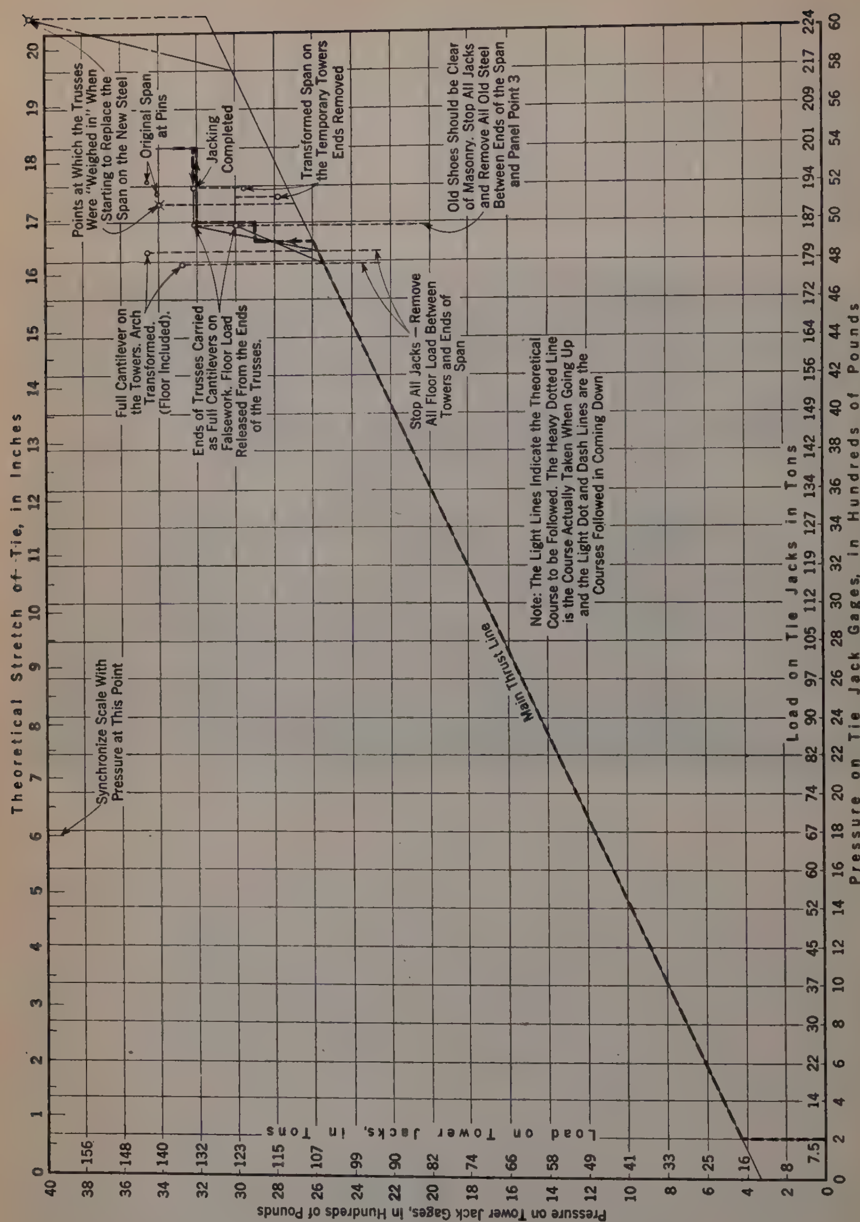


FIG. 5.—TYPICAL CONTROL CHART FOR JACKING OPERATIONS

Handling the structure in this manner required that the ties be designed to take a thrust of 195 tons for each truss, with an allowance for an over-load estimated at 20%, and that each tower be designed to carry a vertical load of 143 tons at each truss. The total weight of the span was 572 tons before removing the old steel. These loads varied somewhat during the process of reconstruction as the removal of old steel and the addition of new members, removal of the floor system, etc., altered the weight of the span with a corresponding variation of the stress in the ties. This is shown in Fig. 5, with an over-load of 18% due to snow and ice, and the shifting of the old pavement (which was torn up and piled along the up-stream truss during the progress of the work).

The difficulty of taking these reactions on the trusses at points where such severe concentrations were not provided for in the original design presented somewhat of a problem, especially because provision had to be made for jacking devices to take up the stretch in the cable ties, and any temporary members used would have to be inserted without disturbing the structure, or without impairing the strength of the members carrying stress. This was accomplished by inserting a system of temporary members in the trusses at each tower, as shown in Fig. 6. A tower member, *A O C D*, was used to carry the vertical reactions and to intersect with the thrust member *O B*, on the thrust line, at Point *O*. The tie stress was then introduced at Point *O* by placing the jacking devices back of this point so that the line of action of the tie would pass through Point *O*.

During the jacking operations the stresses were introduced into the tie and the tower member by picking the proper increment of stress in each so as to keep the resultant stress coincident with Line *O B* as nearly as possible and tangent to the thrust line at this point. The stress from the thrust member was taken up at Point *B* by two distributing members, *B A* and *B C*, and delivered to the upper and lower chords at Points *A* and *C*. As shown in Fig. 3 a brace was used to steady the base of the tower member at the New Hampshire end but this was so arranged that it served no other purpose.

All of the new steel was completely fabricated in the shop. The temporary members were designed so that they could be erected by parts without impairing the strength of the members in the structure to which they were to be connected, except for the removal of a pair of lacing bars in each plane of lacing on the top and bottom chords. Field connections were drilled in the old members, either to a template or to match the new steel blocked in place on the falsework. Main connections were fully riveted and secondary connections and braces were bolted.

For each tie a pair of $2\frac{1}{2}$-in. galvanized bridge cables (6 by 19 on a 7 by 7 core, pre-stretched) were used to straddle each truss and were supported throughout the span on 2 in. by 6 in. wooden cross arms bolted to alternate hangers. These cables had an ultimate strength of 305 tons each and were pre-stretched to 120 tons, approximately 20% greater than the calculated working load. The modulus of elasticity of these cables after pre-stretching was 19 600 000 lb per sq in., for loads of less than 130 tons. Because of their relatively light weight in proportion to their strength, their flexibility, the ease with

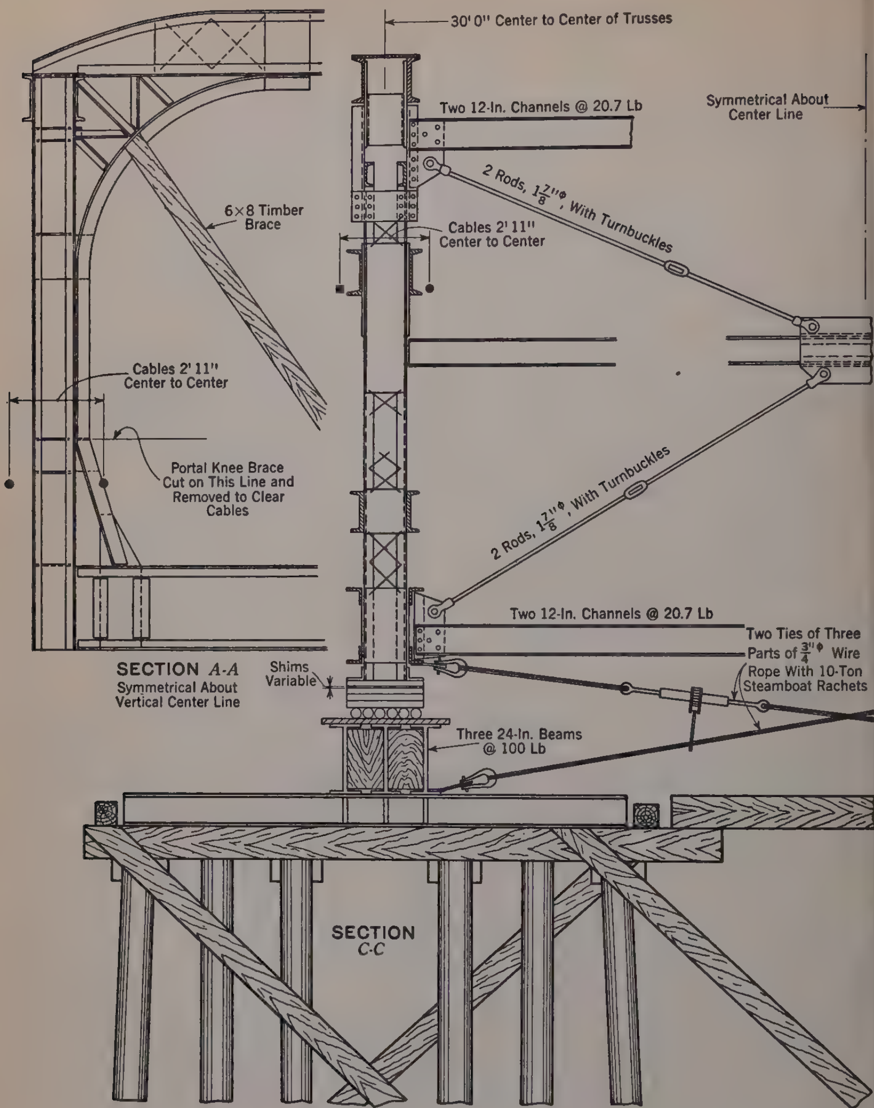
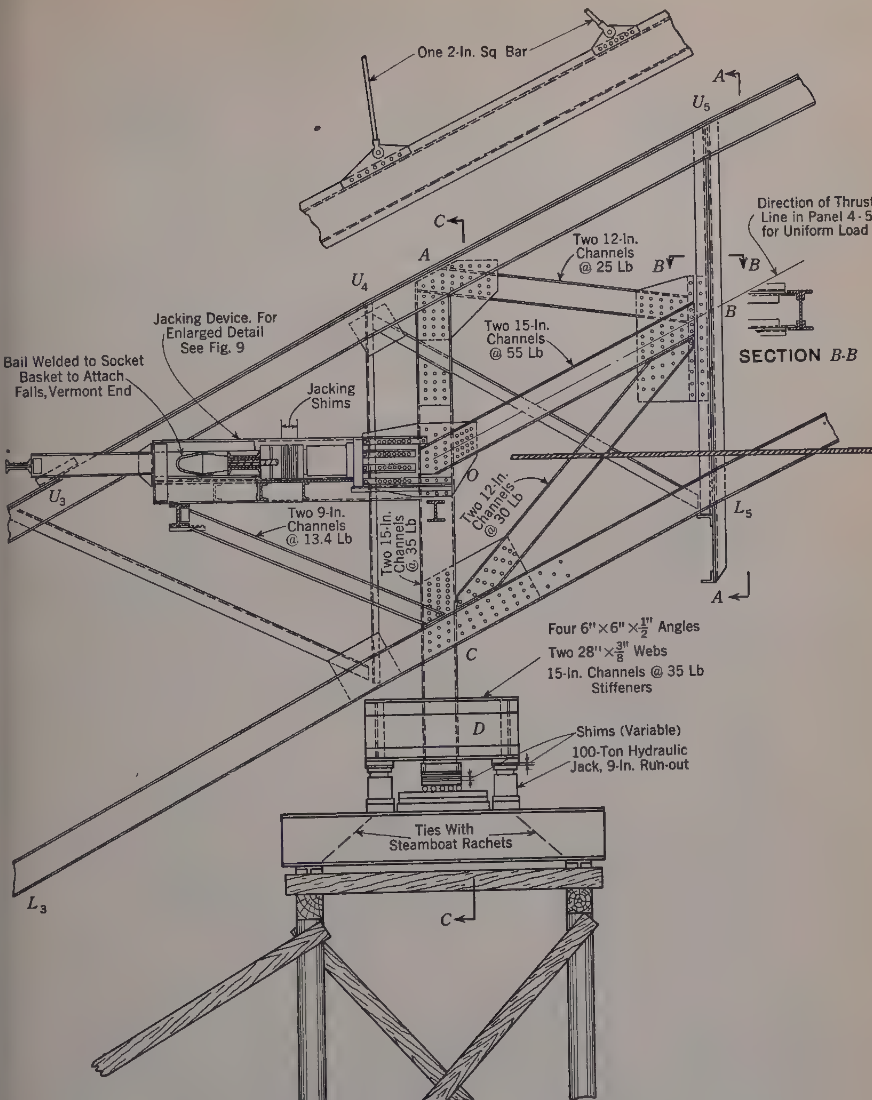


FIG. 6.—TOWER AND

which they could be erected, and their adaptability for connecting to the jacking beams, these cables were particularly well suited for the purpose. They were cut to a carefully measured length and the sockets were added at the mill so that all cables would have the same length when subjected to the same stress. They were pulled into position in the jacking beams to an initial stress of about 5 tons each and their length under this stress was checked at



Jacking Layout

the mill to make certain that adequate clearance would be provided in the jacking devices to accommodate both the stretch and sag.

Jacking Operations

The general plan of the cable and jacking device layout is shown in Fig. 7, the cable lengths being as follows:

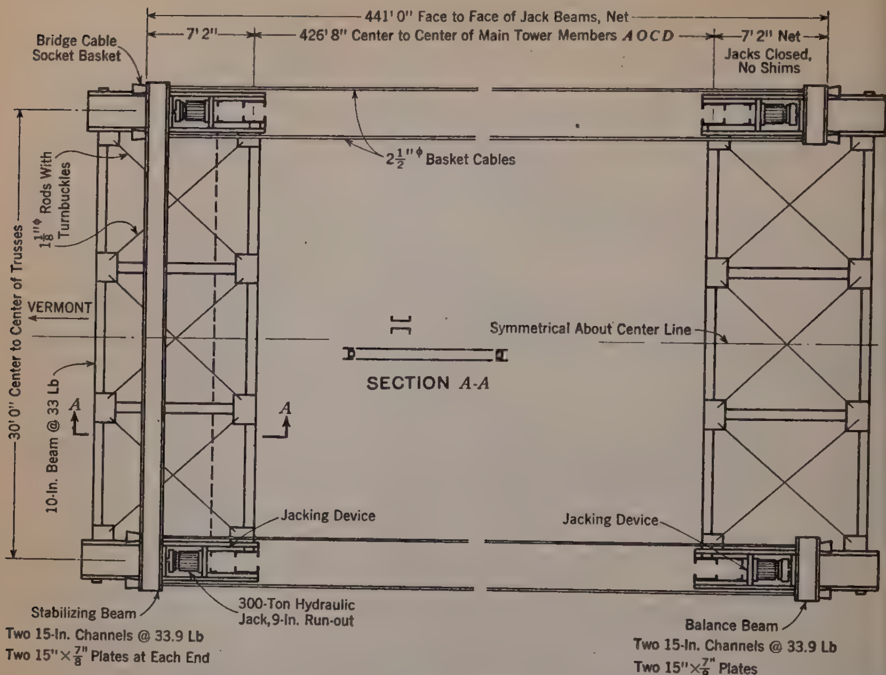


FIG. 7.—LAYOUT OF JACKING DEVICE, SHOWING ARRANGEMENT OF JACK BEAMS AND STABILIZING BRACES

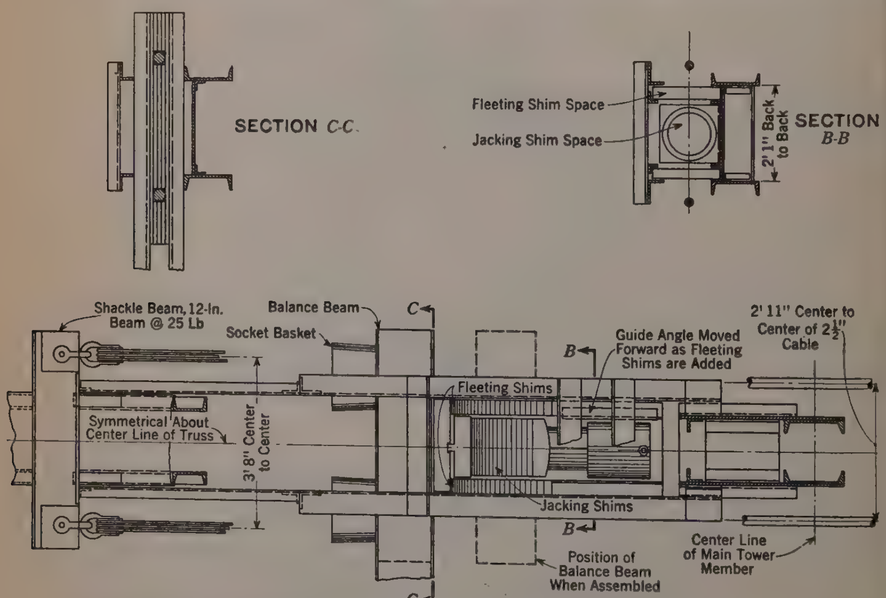


FIG. 8.—PLAN OF JACKING DEVICE, SHOWING PART PLAN OF FALLS USED TO STRETCH THE CABLES INTO PLACE

Distance from center to center of main towers...		426 ft 8 in.
Length of each jacking device from the face of jacking beam to the center line of the main tower (7 ft 2 in. times 2)................		14 ft 4 in.
Net distance, face to face of beams............		441 ft 0 in.
Allowance for sag, cable supported 27 ft on centers, or at cross arms....................	$6\frac{1}{2}$ in.	$6\frac{1}{2}$ in.
Allowance for distortion of truss due to buckling of bottom chord........................	3 in.	3 in.
Provision for normal shims....................	6 in.	6 in.
Gross length of cable at 10 000-lb stress........	1 ft $3\frac{1}{2}$ in.	442 ft $3\frac{1}{2}$ in.
Stretch of cable (calculated); load, 10 000 to 200 000 pounds...........................	1 ft $4\frac{1}{2}$ in.	
Marginal jacking capacity 9 in. at each end.....	1 ft 6 in.	
Total jacking capacity, two jacking devices..	4 ft 2 in.	

The tension in each tie was equalized between the two cables by means of a short balance beam to which the cables were attached at one end of each truss, and the system was stabilized by means of a beam extending across the roadway from one truss to the other. The other end of all cables was attached at this beam. These beams (also referred to as jacking beams) were all free to rotate about a vertical axis, on the center line of each truss, by resting on seats in the jacking devices, provided with knife-edge bearings. This system permitted the independent movement of the trusses so that one could be moved longitudinally, or one tie length could be varied without affecting the other when supporting the temporary towers. The jacking devices being in compression, it was necessary that they be stabilized by means of some kind of bracing system. This was provided as shown in Fig. 7, using diagonal braces composed of rods fitted with turnbuckles and clevises so that the rods could be let out or taken up as might be required to rack the bracing.

Because of the limited clearance for the jacking beams used to equalize and stabilize the cables, it was necessary to provide two jacking devices, one at either end of each tie so that the tie could be taken up or let out at either end. However, both jacking devices were not to be used simultaneously. One 300-ton jack with a 9-in. run-out was used in each jacking device, and shims were provided for fleeting the jacks as well as for carrying the load independently when the jacks were not actually in operation. Fig. 8 shows the detail of the jacking device and, with Fig. 9, shows the assembly of the jack beams and the method of attaching the cables to the beams. Both cables were pulled into place at once, with the falls attached to socket baskets, a bail being welded on them. The falls were set 3 ft 8 in. apart so that the socket baskets would clear the jacking device. When the cables were pulled into

FIG. 9.—DETAIL VIEW OF THE JACKING DEVICE AND THE ANCHORAGE OF THE CABLES TO THE STABILIZING BEAM

FIG. 10.—VIEW OF DISTRIBUTING MEMBERS, WITH THE JACKING DEVICE REMOVED TO SHOW THE JACK SEAT STIFFENERS.

position, they were drawn together to a spacing of 2 ft 11 in. and the remaining beams were assembled in place. Fig. 10 shows the jacking device removed to give a view of the stiffeners and stiffener plates on which the base slab supporting the jacks was seated.

The arrangement of the jacks and shims on the towers is shown in Fig. 6. Four 100-ton hydraulic jacks, also having a 9-in. run-out, were used on each tower, two under each truss. A built-up box girder was provided to carry the load from the jacks to the tower member (see *A O C D* in Fig. 6). The tower member was framed through the girder and was fitted with a base slab to rest on a set of shims when the jacks were not in operation. To provide for movement of the span on the towers these shims were carried on a double nest of rollers so that movement could be obtained both longitudinally and laterally. Longitudinal movement was governed by the jacks in the main ties and lateral movement was controlled by a system of "steamboat" ratchets, shown as a part of the bracing in the plane of the tower in Fig. 6. All diagonal bracing in the towers, except that in which the ratchets were used, was composed of rods with turnbuckles and clevises for adjustment when aligning the span.

Each of the temporary wooden towers was composed of two bents of twelve piles each. The position of all piles, particularly those in the bents forming the approach for the traveler from the shore, had to be located very carefully to avoid interference with the old steel and the emergency bracing, or with the new members. The main towers were braced rigidly and the services of a diver were required to place the bracing below water level. All piles, including the steel piles used in the permanent construction, were driven with a double-acting steam hammer (No. 9-B-2), in swinging leads handled from the boom of the traveler, the operator working out from the shore on the temporary bents. These bents also served to support the old floor system at times during the progress of the work. The old floor system over the new work was not removed completely until the span was almost fully transformed into a tied arch and carried on the temporary towers. The purpose of this procedure was to avoid disturbance to the structure, or any changes in the condition of loading, until the span was well under control.

Jacking operations on major bridge structures are quite common but they are normally conducted under circumstances such that the conditions of stress in the structure are known, and they may be kept under control throughout the procedure. In the case of the jacking operations, in transforming this arch span, the engineers and erectors were confronted with the problem of beginning to operate jacks on a structure in which the stresses were mostly unknown and with many of the members manifestly over-stressed. A jacking load applied in the wrong position or in the wrong direction, or the successive applications of the loads in the wrong order, might easily have caused a failure in some over-stressed member.

The general layout of the jacking equipment is shown in Fig. 11. All pipe, tubing and fittings were ½-in. double, extra-strong stock. All pumps, gages, and central control valves were placed in a group on the floor of the bridge and connected to the jacks at different elevations, by copper tubing.

Emergency stop-and-release valves were placed in the lines, at the jacks. The tie jacks and the tower jacks in pairs, were placed on separate pipe lines and each line was connected so as to operate normally from a separate hand pump. However, the hook-up was so arranged that both pumps might be "cut in" on either line by means of the stop-and-release valves; but, as the two lines worked under different pressures, they could not operate together.

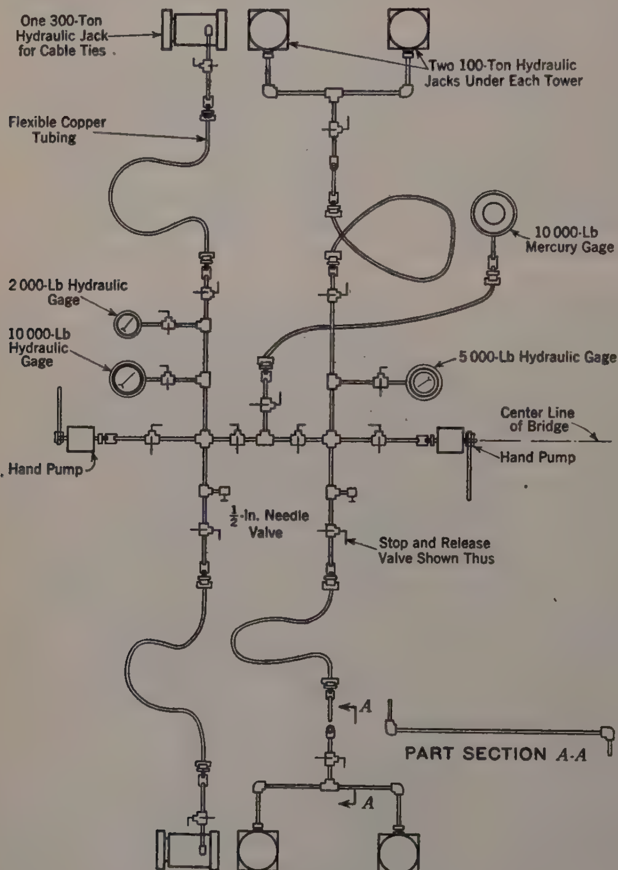


Fig. 11.—Layout of Hydraulic Jacking System

Separate hydraulic gages were used on each line to control the pressure in the jacks; 5 000-lb gages were used with the tower jacks, and 10 000-lb gages in combination with 2 000-lb gages (for low pressures) were used with the tie jacks. Mercury gages (10 000-lb pressure capacity) were connected into the system at either end of the span for use as master gages in the event any discrepancies appeared in the readings of the hydraulic gages. However, it was not necessary to use the master gages because the others had been adjusted and calibrated at the tool house before they were shipped to the field, and they

worked perfectly in every way. Their sensitiveness was very apparent on the gages governing the stress in the ties where the slightest variation in pressure on the jacks, as indicated by the gages, was instantly discernible on the yardsticks clamped in the jacking devices to measure the stretch in the ties. All pumps and gages at either end of the span were placed in a group on the floor of the bridge. There was no hydraulic connection between the jacking system at one end of the span and that at the other; but a telephone system fitted with head-sets was provided as a means of co-ordinating the work. A foreman was placed in charge of the jacking equipment at each end; and, with the gages arranged so that he could read the jack pressure at a glance, he applied the pressure as directed by the engineer in charge, or the superintendent, who directed the operations by working from jacking charts prepared in advance (see Fig. 5).

Two jacking charts were required, one for each end of the span. Differences in the weight localized over the towers—such as the temporary steel, removal of part of the floor system, sand ballast, etc.—necessitated somewhat greater working pressure in the tower jacks under the New Hampshire end. Fig. 5 is the chart for the Vermont end only; the other, although different, was essentially the same so far as this description is concerned.

In starting to transfer the weight of the span to the falsework, the temporary steel (which weighed approximately 26 tons and which had been carried on the falsework by shimming up tight under the foot of the temporary tower members shown as $A\ O\ C\ D$ in Fig. 6) was first to be picked up on the tower jacks at 350 lb pressure. The initial stress on the ties (5 tons on each cable) corresponded to a gage pressure of 260 lb on the tie jacks. Actually the initial tie stress was less than 5 tons per cable and the load on the tie jacks was picked up at 200 lb pressure. According to the jacking chart this initial pressure on the tie jacks called for a total pressure on the tower jacks of 410 lb. The first step then, having taken the tie load on the jacks at one end, was to raise the pressure in the tower jacks to 410 lb at both ends of the span, working simultaneously. Both trusses were jacked in unison from the start, with the corresponding jacks in each truss "cut in" on the same line. This first step placed the entire jacking system "on the chart." Thence using the tie stress to govern the tower reactions in accordance with the chart, the tie stress was stepped up by 200-lb increments of pressure alternating with the tower jacks and stepping up the latter simultaneously at all points. Thus, with the tie jacks at 200 lb and the tower jacks at 410 lb the ties were stepped up to 400 lb while the tower jacks "stood by." The towers were then stepped up to 505 lb while the tie jacks "stood by." This process was repeated until a pressure of 2 600 lb was obtained on the tie jacks, corresponding to a tension of 97 tons in each tie with a total stretch of $8\frac{3}{4}$ in. on the cables. To keep the stretch in the ties about equally divided between the two ends of the span the jacking operations on the ties were then switched to the other end of the structure and the process was continued as before until a pressure of 4 780 lb on the tie jacks was obtained. This pressure corresponded to a stress of 178 tons on each tie, or 89 tons on each cable. This was the limit of the jacking that should be done, according to the calculated reactions, by following the direction of the thrust line on the chart, because it was the maximum stress obtainable in the

tie with the structure resting on the towers and with the ends cantilevered under full floor load. During the jacking to this stage the sand and gravel ballast that was placed over the haunches of the up-stream truss, at the New Hampshire end, was gradually removed and the nuts were removed from the old anchor bolts in the skew-backs.

In this stage the increment of pressure on the towers jacks was approximately 95 lb, which corresponded to an increase of load of 4 tons on the tower. In effect this amounted to a concentration of 4 tons on each truss at each of the towers. It was thought that the most seriously over-stressed members in the structure, before any jacking was done, were the diagonals, U_2-L_3 and U_3-L_4, and the verticals U_2-L_3 (see Fig. 3). These diagonals were in compression and the verticals were in tension. To jack upward on the tower member (that is, to apply the increment of the tower reaction before the increment of tie stress was applied) would operate to increase the stress in these members; but, applying the tie-stress increment first, and then applying the tower-load increment, would serve to reduce these stresses. Therefore, this was the procedure adopted.

When the stress of 180 tons on the tie was obtained, all jacking was stopped and the floor deck removed; part of the steel work in the floor system was left supported on the temporary pile bents, but all floor load was removed from the first three panels at each end of the span. . The approach span at the Vermont end was also supported on falsework at the east end to permit the removal of the old shoes a short time later.

Referring to the jacking chart in Fig. 5, it will be observed that there are two positions plotted for the positions of the jack pressures after departing from the main thrust line: One is for the maximum, and the other is for the minimum, calculated dead weight of the span. When jacking was resumed after the floor loads were relieved, the tie jacks were "stepped up" by increments of 100 lb of pressure, each being followed by a "step-up" of the tower jacks by an increment of 300 lb of pressure. After one "step-up" on the tie and one "step-up" on the towers the tie stress was determined as at 4 980 lb per sq in. jack pressure, a total of 185 tons. At this tie stress the chart indicates that the old shoes should be clear of the masonry skew-backs. An inspection of the shoes failed to indicate that this was so, and the tie jacks were once more stepped slowly to a jack pressure of 5 000 lb per sq in. The tower jacks were then "stepped up" slowly and when the pressure in these jacks reached 3 200 lb per sq. in., an observer stationed at the down-stream shoe, at the New Hampshire end, reported that there was a very definite crack in the ice and grout around that shoe but no change could be observed at the other shoes. The ties were then stepped up to 5 200-lb jack pressure and it was observed that the joint U_3 on the up-stream truss (New Hampshire end) had opened about $\frac{1}{16}$-in. This joint, as well as all others at which the new steel was to be connected to the old, had been prepared previously for dismantling by replacing the rivets with bolts. The down-stream jacks all around were then cut off the lines and the up-stream truss was stepped up to 5 400 lb per sq in. (201 tons), on the tie, and to 3 300 lb per sq in. (136 tons), on the tower, and then returned to the previous position corresponding to the down-stream truss. This was simply a

test movement. Movements at the joints that were bolted indicated that the members were carrying no stress of any importance, and consequently the stress in the trusses between the jacking devices and the ends of the span was released. The total stretch in the ties at this point was observed to be $17\frac{5}{8}$ in.; and the total rise in the span directly over each tower, as determined by taking elevations on the floor beams at these points, was found to be $\frac{7}{8}$ in. The span was landed on shims in this position and the jacks released. The nuts were placed back on the anchor bolts at the Vermont end so as to stabilize the structure on the falsework and the old steel was completely removed from the New Hampshire end.

The total jacking time required for this work was about 12 hr including the time required for testing the jacking system for leaks, etc., and for checking the gages, one against the others. During the first stage of the jacking, which increased the tie stress to 178 tons, the weather conditions were very inclement. This stage of the work was begun at 8:00 A.M. on November 10, 1936. The day was quite cold, the temperature being approximately 32° F, and soon after the work was started it began to rain, and continued to rain and sleet until noon. After noon a high wind rose and continued for the remainder of the day. This made progress with the jacking very slow so that by evening the first stage of it was only about half completed. Work was continued the following morning; the weather was clear and the temperature was approximately 20° F, but it was much more favorable than the day before. The remainder of the jacking necessary to bring the tie stress up to 180 tons was completed in about $2\frac{1}{2}$ hr. After about a day required to remove the floor load from the ends of the span the jacking was completed in approximately 1.5 hr. Under favorable weather conditions the entire jacking operations would have been completed in about 7 hr of actual jacking time.

After the removal of the old steel at the New Hampshire end, the jacks on the temporary tower at the same end were raised about 15 in., rotating the span vertically about the old pins at the Vermont end and, of course, raising the jacks at the Vermont tower proportionately. The tie jacks at the New Hampshire end of the span were also "taken up" (that is, the ties were shortened) about 5 in. on the up-stream truss and about 3 in. on the down-stream truss. The tower jacks on the up-stream truss, New Hampshire end, were then lowered about 2 in. and the bracing rods in the tower were adjusted at the same time by means of the turnbuckles. The difference of 2 in. in the distance that the ties were shortened relative to each other removed most of the sag in the up-stream truss and almost straightened the span vertically, bringing it back almost plumb. The actual pressures on the jacks during this set of operations naturally did not change except that, in the case of the tie jacks, the tie stresses increased about 10 tons to overcome the roller friction. The New Hampshire end was free and it was simply moved to make clearance for the erection of the new steel. At this time also the span was moved down stream 2.5 in. on the New Hampshire tower and almost 4.5 in. on the Vermont tower by means of the steamboat ratchets shown in the lower stage of bracing in Fig. 6. This jacking, together with the adjustment of various turnbuckles in the temporary rod bracing, was done simultaneously with the jacking to

"take up" on the ties. Otherwise it is doubtful whether the span could have been moved so far laterally on the Vermont tower.

Steel Erection

Before removing the old steel the spring-line pins were referenced carefully and the center line of the structure was established on the towers to correspond to the skew-backs. There was a rumor locally that, during high water in 1927, a wash-out had occurred back of the New Hampshire abutment, causing it to settle. Careful measurements were made to determine the relative positions of the pins all around, and levels were taken to check their elevations. It was found that the old pins were within $\frac{1}{16}$ in. of the same elevation. The nominal span was 540 ft, center to center of the old pins, but the measurements taken indicated the up-stream truss pins to be 540.05 ft and the down-stream pins to be 539.91 ft, center to center—a difference of $1\frac{3}{4}$ in. The measurements also showed that this difference was divided between the two abutments—$\frac{1}{2}$ in. to the New Hampshire abutment and $1\frac{1}{4}$ in. to the Vermont abutment. This finding seemed to indicate that each abutment had rotated about a vertical axis causing a change of span for both trusses. The up-stream truss was lengthened $\frac{5}{8}$ in. in span and the down-stream truss was shortened $1\frac{1}{8}$ in. This seemed rather improbable since there were no cracks in the wing walls nor any other visible indications of settlement; and, since the New Hampshire abutment (which was supposed to have been the one affected) had the least skew, no further attention was paid to the settlement rumor. However, the new steel was set to "square up" with the center line of the structure by working from the referenced position of the old pins in the down-stream truss to establish the length of the reconstructed span according to it. The up-stream truss span was shortened $1\frac{3}{4}$ in. in this manner.

Before placing the new main members the light-welded steel trusses shown in Fig. 12 were shored and blocked in place to serve as guides to hold the steel piles in exact position while driving. These trusses were designed for this purpose as well as to serve as reinforcement for the concrete piers. The weight of the piers was assumed to be carried by the piles entirely, without any other support, although the rip-rap, on which the piers were placed and through which the piles were driven, was washed off and cleaned of mud, etc., by a diver using a fire hose. The timber crib shown in Section *A–A*, Fig. 12, had been filled with rock and capped with concrete some time after the original span was completed. The rip-rap apron around the old crib was cleared away where necessary and 12 in. by 12 in. timbers were cut away to make room for the new steel piles. The piles were driven to a penetration of about 65 ft to rock and were capped with steel grillages to support the forward shoes of the new steel work. The shoes were set in position carefully on the grillages, to measurements from the referenced position of the old pins. The wood sheet-piling was then driven to provide forms, and the concrete placed by means of a tremie. Anchor bolts 3.5 in. in diameter, two for each shoe, were placed in the shoes and the concrete was poured to the bottom of the grillages. The positions of the shoes were checked, the nuts on the anchor bolts locked down tight,

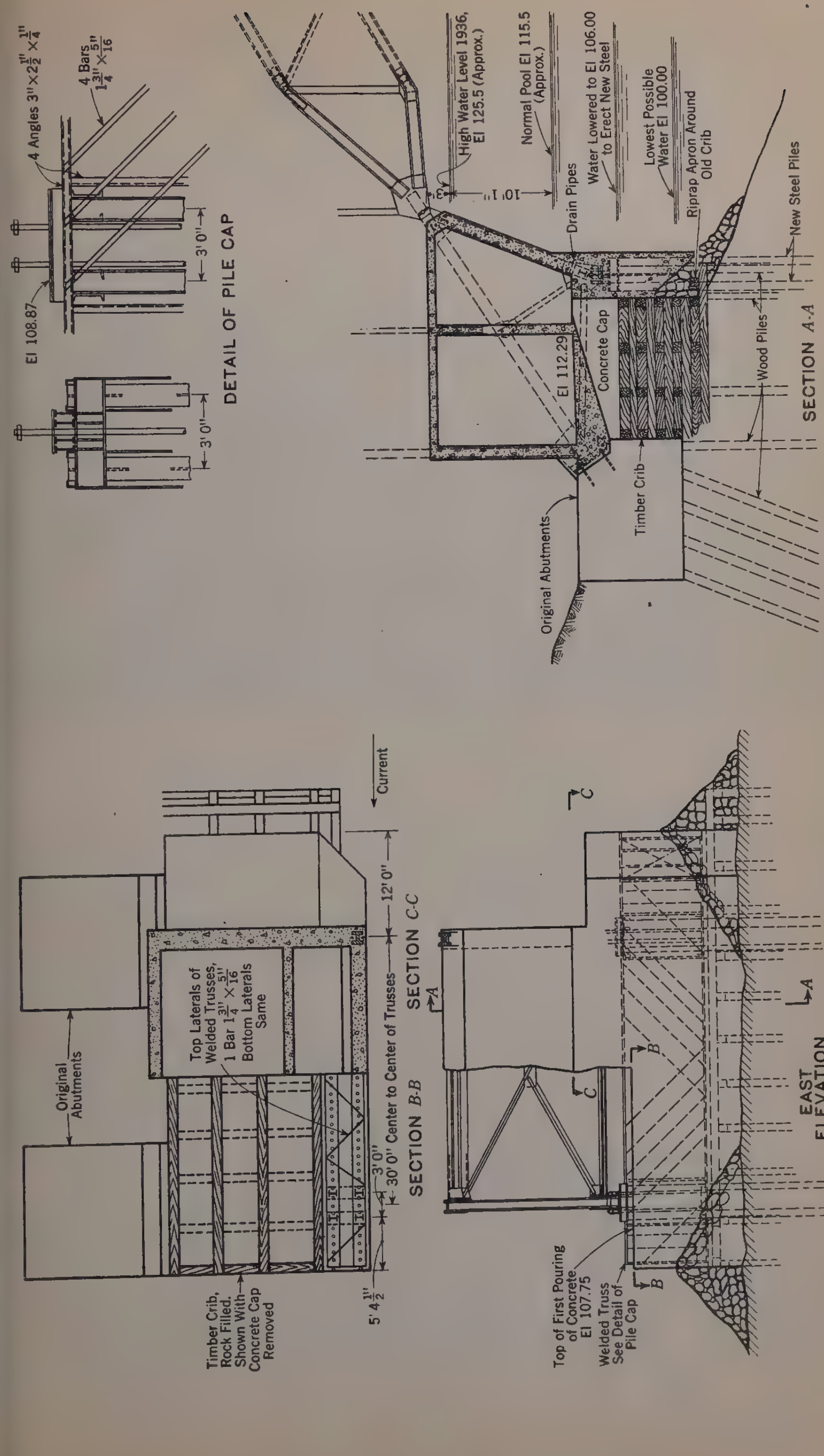


Fig. 12.—Construction Layout of the Vermont Pier

the steel below the new pins erected, leveled up and shimmed in place, and the skew-back shoes which were fabricated on the main thrust member were grouted in position. The old anchor bolts were left in the skew-backs and were used to provide anchorage for the new steel. The new members were then riveted up completely and the concrete was poured to the floor level in the concrete encasement shown in Fig. 12. The calculated loading on these steel pile groups which consisted of four, 12 in. W F, 55 lb sections was:

Load per pile:	With wind	Without wind
Before placing concrete	+ 4 to − 47	− 5 to − 38
After all concrete was placed	− 67	− 58

The new steel above the pin was erected independently of that below, in cantilever fashion, by connecting at the old splices on both the top and bottom chords which were placed conveniently just above Joints U3 and L3 (see Fig. 3). This is shown in Fig. 13.

After the steel at the New Hampshire end of the span was erected, the tower jacks were released to lower the New Hampshire end by rotating about the Vermont pin; and the tie jacks were "let out," to make the pin holes in the new steel match. The pins were then driven and the old steel at the Vermont end was cut away immediately and removed. The span was then raised about 15 in. at the Vermont tower by rotating about the New Hampshire pin and the cables taken up again to make clearance for the new erection as had been done at the opposite end of the span. To complete the straightening-up process, the span was moved down-stream approximately $\frac{1}{2}$ in. farther at the Vermont tower, the up-stream truss was dropped about 2 in. at this point, and the tower bracing adjusted.

The same procedure was followed in the erection of the steel, and in the other construction at this end of the structure, as was used at the other end. When the erection was completed and ready to be connected, the loads at all towers and the stresses on the ties were picked up on the jacks and "weighed in," one truss at a time. The jack pressures were then plotted on the charts (see Fig. 5). The up-stream tie jacks picked up the stress on the tie at 6 100 lb per sq in. (228 tons); the up-stream tower jacks picked up the load at 4 100 lb per sq in. (168 tons); the tie jacks in the down-stream truss picked up at 5 100 lb per sq in. (190 tons); and, the tower jacks under this truss picked up at 3 400 lb per sq in. (140 tons). Due to the fact that several inches of snow and ice covered the deck of the span at this time and during the previous month, the old asphalt plank flooring had been piled along the up-stream truss, permitting the old plank sub-floor to become thoroughly soaked with water. The pressures at which the jacks picked up the loads were considerably higher than those required to raise the span at the beginning.

After the loads were weighed in on the jacks, the span was lowered, rotating about the New Hampshire pin, and the ties were released slowly so as to bring the pin holes into position for driving. The jack pressures did not change during this operation so that as the pins were driven the pressures in the jacks were already charted.

As soon as the pins were driven the structure was lowered immediately and landed on the abutments. Since the up-stream truss was the heavier of the

two, it was started down ahead of the other and the pressures in the jacks controlling it were lowered to meet those of the jacks controlling the down-stream truss. The corresponding jacks on the two trusses were then thrown in on the same lines and the two trusses were thence brought down in unison. Thus, the down-stream jacks were "cut off" the lines but were held under pressure, and the tower jacks at both ends of the up-stream truss were lowered

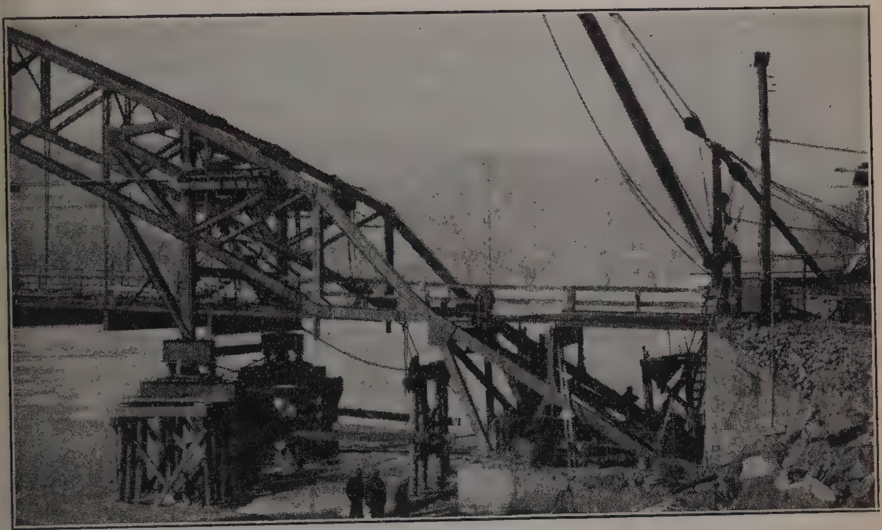


FIG. 13.—NEW STEEL ERECTED AT THE NEW HAMPSHIRE END OF THE SPAN READY FOR DRIVING THE PINS

simultaneously by opening up a release valve in the line while the tie-jack line was kept closed and the pressure was permitted to "drift" as the tower jacks released the pressure. These jacks were not turned loose but were held under control by "stepping down" at 200-lb increments.

As the jacks were stepped down the ties gave up the load very leisurely, accumulating sag between the supporting cross arms until the original initial stress in the tie was reached. The shims were placed under Tower Members *A O C D* (Fig. 6) to take the local concentration of temporary members away from the trusses, and the release valves in all jacking lines were thrown wide open. The total time required to release the jacks and land the span was a little less than 2 hr.

Throughout the jacking operations in raising the span on to the towers, and again when landing it on the abutments, a peculiar difference between the action of the tower jacks at the two ends was manifest. As the stress in the tie was "stepped up" along the thrust line, the tower jack pressure at the end of the span, where the tie jacks were in operation, had to be pumped to the full increment of pressure, whereas the tower jack pressure at the opposite end drifted automatically in exact accordance with the charts, with little or no pumping after the falsework in the towers had taken up the initial settlement.

At first this was thought to be the result of excessive settlement in the tower where the jacking was required; but after switching the jacking procedure to the opposite end of the ties this same peculiar action continued as before: The jacks on the tower, which had been drifting as stress was jacked into the tie, now had to be jacked the full increment, and the others, which had previously required the full increment of jacking, now drifted with the tie stress. The same perplexing phenomenon continued when landing the span, with no very apparent reason for it.

As soon as the main span was landed, the approach span was also landed back in place by connecting up the bent which supports it, on the shoes of the main span. The temporary steel members were dismantled and removed, and in the meantime the old floor beams and stringers were replaced to make ready for the floor. No concrete was placed around the new steel members above the floor of the new piers, as shown in Fig. 12, but the forms were constructed and made ready for placing the concrete encasing as soon as the span was landed. The old roadway deck which consisted of asphalt plank laid on a 3-in. wood plank sub-floor was replaced with a 3-in. concrete floor and the old wooden sidewalk was replaced with 2-in. floor. The old railing, which consisted of gas pipe, was replaced with a modern form of construction to conform to the State Highway Department standards. Levels taken over the span after the work was completed showed the grade to conform to a uniform curve having a middle ordinate of approximately 18 in. and the floor beams were all practically level, the maximum difference in the elevation of the ends of any floor beam being only $\frac{3}{8}$ in.

Fig. 14.—Falsework at the Vermont End

Working Conditions

The weather conditions at the time this work was in progress were very erratic and generally unfavorable. The late fall and winter seasons began

quite cold and before the first of December the river was frozen over with ice approximately 10 in. thick. About this time it became necessary to lower the pool level to reach the old steel around the skew-backs. This broke the ice along the shores but it caused no serious trouble and the ice quickly sealed up as the cold weather continued. About December 9, with several inches of snow in the Connecticut River basin, the temperature rose suddenly and in three days the precipitation was reported variously as 3 in. to 4 in. Under the effects of this rise in temperature and the rain, the ice broke loose on the river above the pool level and began to move down stream quite rapidly. By the skillful manipulation of the rolling lift gates in the dam the engineers of the power company were able to hold this in check and permit the State forces to construct an ice fender to shield the Vermont tower. Fig. 14 is a view of the Vermont end immediately after the ice had passed down the river. It broke loose about 300 ft above the bridge, and the lower edge of the ice can be seen below the bottom chord of the truss. The ice is still intact around the falsework due to the protection of the fender. Since there was no fender to protect the New Hampshire tower, the gates were kept closed at that end of the dam, to hold the ice and to throw the current to the main channel on the opposite side, discharging the ice past the tower which was protected by the fender. This was so successful that it was decided that it would not be necessary to build a fender to protect the New Hampshire tower.

Conclusion

This project is somewhat novel in the history of arch-bridge construction, as was the design and erection of the original span.[3] The magnitude of this undertaking was by no means measured by the quantity of new material involved; nor was it measured by the cost of the project. The cost of a new structure is scarcely a criterion of the amount of the work incident to the planning and careful procedure which was necessary, and although the final cost was approximately three times the original cost of the superstructure, a new span would have cost several times the amount of the repairs and, in addition, the cost and hazard of removing the old structure would have to be taken into consideration. Because of the depth of the pool at this point and possible damage to adjacent property, the removal of the old span as it stood at the beginning of repair operations would have been quite a problem and undoubtedly would have cost at least half as much as the proposed repairs.

The load capacity of the span is adequate for the location. The arch trusses were found to have capacity for the H-15 loading and the floor system, including the slightly greater weight of the new concrete-steel deck, has a capacity 20% greater than the H-10 loading—that is, the H-12 loading, applied in accordance with the American Association of State Highway Officials Specifications. Therefore, no special alterations were necessary to raise the structure to the traffic requirements. However, when designing the new deck and hand-rail, the roadway was widened from 20 ft to 21.25 ft, and the width of the sidewalk was established at 4 ft 10 in. The new steel and foundations were designed for the H-15 loading, and since the steel in the main trusses is in such excellent condi-

tion that it will probably last indefinitely if properly maintained, it is possible, if future traffic demands it, to revise the entire structure to the H-15 loading at relatively little cost.

On the basis of replacement costs the State Highway Department, by repairing the damaged span, provided an adequate structure to meet the traffic requirements at approximately one-third the cost of removing the old structure and replacing it with a new one of the same capacity. By the use of the specific method adopted to do the work enough was saved to provide for the new deck and hand-rail.

As compared to the original structure, having a span of 540 ft with the spring-line pins 14 ft below present high water, the reconstructed arch has a span of 486 ft with the spring-line pins 3 ft above high water, assuring ample protection against a repetition of the damage at the same high-water level.

This work was done by the engineering and erection forces of the American Bridge Company, who furnished and fabricated the steel, and who furnished the erection equipment, in co-operation with the engineers of the Bridge Division of the New Hampshire Highway Department and their construction forces.

Although all operations were made difficult throughout the work by weather conditions, ice, high water, and extreme cold weather the progress and the final results were very satisfactory to the State Authorities and the bridge is adequate in every way for modern traffic.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

PRELIMINARY DESIGN OF SUSPENSION BRIDGES

Discussion

BY MESSRS. SHORTRIDGE HARDESTY AND HAROLD E. WESSMAN, MEMBERS, AM. SOC. C. E.

SHORTRIDGE HARDESTY,[37] AND HAROLD E. WESSMAN,[38] MEMBERS, AM. SOC. C. E. (by letter).[38a]—The writers greatly appreciate the comments made by those who have discussed this paper. Mr. Ammann emphasizes the major significance of the method when he notes, first, its value as an aid in visualizing the behavior of the unstiffened cables and the effect thereon of the stiffening trusses and other factors; and, secondly, its merit in appraising the economic value of stiffening trusses of various degrees of rigidity. In fact, its value in a variety of economic studies is demonstrated clearly by Messrs. Woodruff and Raab in their interesting discussion.

In the preliminary studies which are necessary for any suspension bridge, it is not only important to have a method which will give approximate maximum moments and deflections quickly at one or two points, so that trusses of different rigidity may be compared, or that other variables may be investigated; but it is even more important that the method permit and facilitate an understanding of the action of all parts of the structure. Here is where the deflection theory, championed by Mr. Moisseiff, displays its shortcomings, for the very nature of the exponential formulas, as is evidenced by even a casual inspection, is such that the related functioning of truss and cable, side span and main span, is completely disguised. In the application of these formulas to the determination of stiffening truss moments, no scale is obtained regarding the relative contributions of each factor to the total moment. In preliminary

NOTE.—The paper by Shortridge Hardesty and Harold E. Wessman, Members, Am. Soc. C. E., was published in January, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: May, 1938, by Messrs. A. W. Fischer, and Jacob Karol; June, 1938, by Messrs. Glenn B. Woodruff and Norman C. Raab; September, 1938, by Messrs. Hardy Cross, and A. A. Eremin; October, 1938, by Messrs. A. Fraser Rose and William A. Rose; November, 1938, by Leon S. Moisseiff, M. Am. Soc. C. E.; and January, 1939, by Messrs. O. H. Ammann and Leon Blog.

[37] Cons. Engr. (Waddell & Hardesty), New York, N. Y.

[38] Prof., Structural Eng., Coll. of Eng., New York Univ., New York, N. Y.

[38a] Received by the Secretary March 16, 1939.

studies, at least, it is important to know how variations in one factor or another will affect final results, in order to obtain an economical design. In commenting about the preliminary design method, Mr. Blog states: "Visualization of the otherwise intricate interaction of all parts of the hybrid assembly has been made easy."

The important studies made by Mr. Karol again emphasize this. Figs. 8, 9, and 10 give an excellent break-down of the contributions of various factors to the moment and deflection at the quarter-point of the main span of the Triborough Bridge.

It is important to note that the writers emphasize the method in its relation to preliminary design studies. It is the preliminary study of any project which calls for the highest degree of engineering judgment, talent, and experience. The final design and its analysis are usually of a routine nature. They demand technical skill, it is true, for their consummation, but they rarely result in major changes in conclusions already reached as a result of thorough preliminary studies. Mr. Fischer, apparently, does not recognize a distinction between, nor the relative importance of, preliminary and final design. Which is more important—making the decision that the stiffening truss of the Bronx-Whitestone Bridge shall be an unprecedented shallow plate girder only 11 ft deep $\left(\frac{1}{210}\text{th of the span length}\right)$—or computing the final curve of maximum moments for the girder as selected?

In making the analysis of the final design, the deflection theory in one form or another should be used. Mr. Moisseiff prefers the deflection theory expressed in exponential formulas, which require a computing machine for accurate results. The writers prefer the deflection theory expressed in terms of a trigonometric series, which gives just as accurate results with an ordinary slide-rule. That is a matter of personal preference.

Mr. Fischer believes that the elastic theory can be used for spans less than 750 ft long. The writers see no merit in using a theory of analysis which is definitely erroneous. They heartily agree with Mr. Moisseiff when he states, "The elastic theory has outlived its usefulness as a tool for designing suspension bridges." Moreover, any preliminary design method based on the elastic theory may lead to distinctly erroneous conclusions, when one realizes that there are a thousand possible stiffening trusses of various rigidities for any suspension bridge. The elastic theory, which ignores cable deflections and dead-load effects, if applied to the Bronx-Whitestone stiffening girders as designed, is 1 300% in error. Other stiffening girders or trusses might have been used in which the error, by using the elastic theory, would be less; but they would be stiffer—and more expensive. A preliminary design method that does not disclose the true structural action is of little value to the engineer who must make his choice primarily in terms of rigidity and economy.

In making economic studies, diagrams, such as Figs. 11 and 12 plotted by Mr. Karol in applying the writers' method, are a distinct help in getting the answer to the following question raised in the paper:

"Is it more economical to obtain stiffness by increasing the chord section or by increasing the depth of the truss?"

In a simple truss, increasing the chord area lowers unit stress. Increasing the depth of truss also lowers stress. On the other hand, if the depth of a stiffening truss is increased, the unit stress is actually increased. If unit stresses are too high, decreasing the depth and increasing the chord area will bring them down; but, at the same time, the effect upon rigidity must be kept in mind.

Messrs. Woodruff and Raab have presented an interesting summary in Table 3 showing the effect of moment of inertia on rigidity, associated with changes in dead load of floor slab and dead load of trusses and cable. It is desirable to keep dead load down in order to save on cable and tower steel. Using a light-weight deck will help to do this, but the light-weight deck and the supporting floor steel may cost more than a standard concrete slab and its framing. Will the saving in one item offset the increase in another item?

Moreover, if a definite rigidity (whatever that may mean) is to be attained, higher dead load is of help in reducing deflections, but it may be cheaper to use a stiffer and more costly stiffening truss to get the same result, in order to save on cables and towers.

Mr. Eremin states that there are various limitations to the writers' method which should be considered by designers, but he fails to point out any limitations of significance.

The writers are indebted to Mr. Moisseiff for calling their attention to the fact that the point of maximum positive moment in the Bronx-Whitestone Bridge is at a point one-tenth of the span length from the tower. The moment at that point (6 360 ft-kips) has been verified by the writers, using the trigonometric series method. On the Golden Gate Bridge, the point of maximum moment is near the eighth-point. It is an interesting fact that, on these two large spans with unusually flexible stiffening units, the point of maximum moment does not occur at or near the quarter-point. That does not nullify the value of the writers' method as an aid in preliminary design studies. As stated by Messrs. Rose and Rose, the method may be extended for the computation of maximum moment at sections other than those given. It is questionable, however, whether this is worth doing for preliminary studies, although the point is one that warrants additional investigation. In making initial designs, it is advisable to correlate stiffening truss moment at some section with deflections at the same section, in order to get a scale on rigidity, as indicated by Fig. 7.

Fig. 15 has been plotted to show how the unstiffened cable deflects under an advancing uniform live load with the cable length kept constant. Note that the maximum deflection occurs almost at the quarter-point with live load extending over a distance of $0.4\,l$ into the main span. A load extending about $0.2\,l$ into the main span causes a loop with maximum deflection near the eighth-point, but the deflection is considerably less than the maximum at the quarter-point. When cable-stretch effects are included, the difference between the two becomes even greater. It was because of this, and also because maximum positive moments on a large number of spans already designed occurred at or near the quarter-point, that the writers developed the preliminary design method primarily in terms of quarter-point effects.

Fig. 15 indicates the possibility of further interesting studies on suspension bridges. It is not the deflection, but the curvature of the curve of deflection of the cable which is important in evaluating stiffening truss moments. The

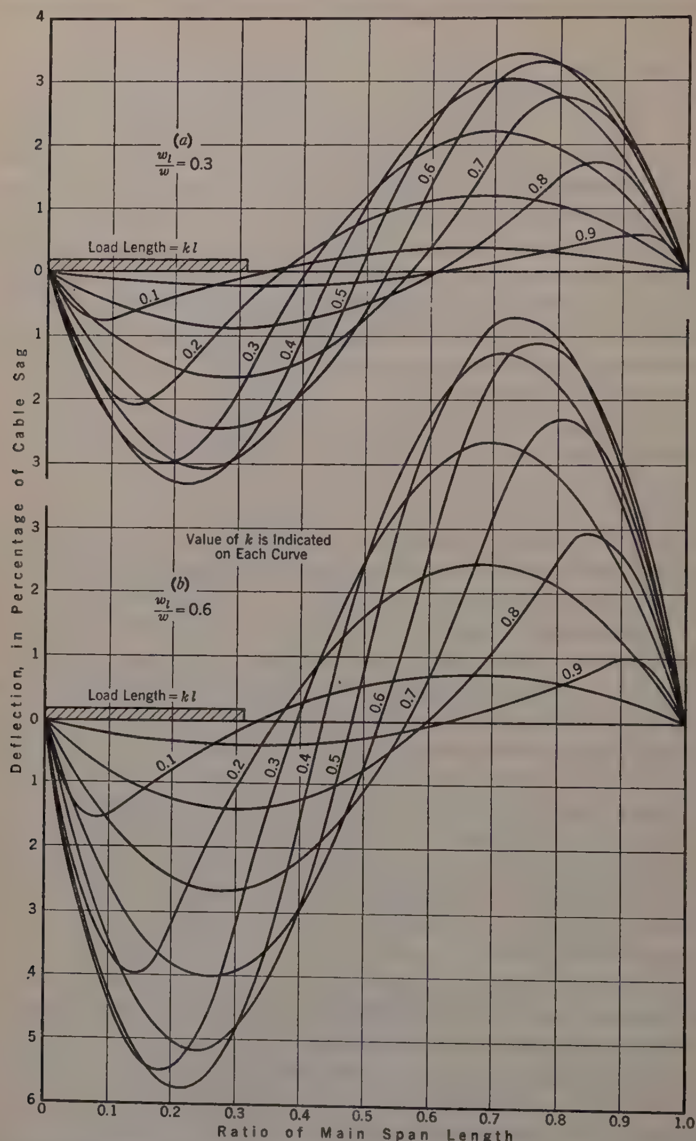


FIG. 15.—DEFLECTIONS OF UNSTIFFENED CABLE DUE TO PARTIAL LIVE LOAD OF VARYING LENGTH EXTENDING FROM LEFT END OF MAIN SPAN. DEAD LOAD OVER ENTIRE SPAN

stiffening truss must have the same curvature as the curve of deflection of the cable, neglecting suspender deformations.

The differential equation of curvature for the cable given in Equation (21*c*)

may be restated, as follows:

$$\frac{d^2\eta}{dx^2} = -\frac{w_s - \beta w}{H_a + H_w} \quad \text{(50)}$$

The differential equation of curvature for the stiffening truss is:

$$\frac{d^2\eta}{dx^2} = -\frac{M}{E I} \quad \text{(22)}$$

In the unstiffened cable, an inspection of Equation (50) indicates that the curvature of the curve of deflection of the cable, with cable-stretch effects omitted, may be slightly greater at the eighth-point than at the quarter-point. It is dependent somewhat on the ratio of live to dead load. The denominator decreases if H_a (live load horizontal component of cable stress) becomes less. The numerator will increase, because βw decreases.

When a flexible stiffening truss, such as that of the Bronx-Whitestone Bridge, is added to the cable, the stiffening effect is somewhat localized in nature. The moment corresponding to Step I of the preliminary design method is then greater at the eighth-point than at the quarter-point. The quantity $w_s - \beta w$ (which is the suspender pull minus a fractional part of dead load) is directly related to the moment, as evidenced by Equations (50) and (22). A study of Fig. 6 clarifies this relationship.

The moment from Step II, however, involving cable-stretch effects, is definitely greater at the quarter-point than at the eighth-point. As noted under the heading, "Effect of Cable Stretch, Temperature Change, and Side-Span Interaction: Effect of a Change in Sag Upon the Main-Span Stiffening Truss," the moment diagram is elliptic in shape rather than parabolic, with the maximum moment at the center of the span. As flexibility increases, however, the moment due to Step II becomes a smaller part of the total, as shown in Fig. 9. Consequently, even if the moment from Step II is greater at the quarter-point than at the eighth-point, the difference is not as great as the difference in Step I where the effect at the eighth-point predominates. Hence, the point of absolute maximum positive moment may have a considerable range depending upon the "stiffness" of the stiffening truss. A detailed study of characteristics of maximum moment curves would be of some value, in view of the tendency toward more flexible stiffening units. It is quite possible that the maximum moments in some cases may be almost constant over that part of the span, say between the tenth- and the fifth-point. This is an added argument supporting the tendency to use virtually the same section throughout for the stiffening girder or truss, a practice which, of course, appeals to the fabricator.

Mr. Moisseiff undoubtedly recognizes that, as one gets closer to the tower, the effect of suspender elongation, and the effect due to the location of the end pin of the stiffening girder not being in the same vertical line as the end of the cable above, upset considerably the moments and shears derived by the usual deflection theory formulas.

Messrs. Rose and Rose have submitted a valuable discussion, linking the suspension bridge to the classification of structures proposed by Professor

Cross,[39] a classification which is of tremendous importance to the profession. When its significance becomes more widely recognized, practicing engineers will design structures even more intelligently than they do now; teachers of indeterminate structures will be more careful in relating procedures of analysis to design.

The writers are also indebted to Messrs. Rose and Rose for calling attention to the relatively unimportant part played by shears in preliminary design studies. Incidentally, Mr. Karol notes that he has extended the method to account for maximum shears. It is important to note again that maximum end shear cannot be determined accurately by any theory that ignores the location of the end pins of stiffening trusses relative to cable.

Mr. Blog submits a simplification of the method which is quite acceptable. It will undoubtedly save time to eliminate the small elements of length denoted by L', and to work with all initial dimensions referred to the center line of the tower. Table 5 indicates that the results obtained by this simplification are accurate enough for preliminary studies.

Professor Cross draws upon the structural action of the flitched beam in clarifying the picture of the interaction of cable and stiffening truss. It is also important to keep Fig. 6 in mind in picturing the load distribution between the two elements. He also expresses the hope that more attention will be given to the function of the stiffening truss, with a view to defining stiffness adequately. There is undoubtedly a definite need for further research in this field, a need that challenges the resourceful investigator. The problem is not a simple one.

This is further emphasized by Mr. Ammann in his enlightening discussion concerning rigidity and recent trends toward more flexible trusses. The writers are indebted to him for noting that the factors which influence the design of the stiffening system are so complex that the trial method of approaching a final design, combined with judgment based upon experience, is less likely to lead to erroneous practice than "cut and dried" formulas or rules of design.

Correction for *Transactions:* Change Equations (33*b*) and (33*c*) to read,

$$df_1 = -\frac{3}{16\,n_1}\sec^2\alpha_1\,dl_1 \dots\dots\dots\dots (33b)$$

and,

$$dl_1 = dL_1 \sec\alpha_1 \dots\dots\dots\dots (33c)$$

[As printed in *Proceedings*, Equations (33*b*) and (33*c*) were derived assuming that α_1 remained constant. It is more nearly in line with actual conditions to assume that the elevation of the tower top is unchanged during a horizontal movement. Equation (34), and the corrected Equations (33*b*) and (33*c*), were derived on that basis.]

[39] "The Relation of Analysis to Structural Design," by Hardy Cross, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 1363.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

ENGINEERING ECONOMICS AND PUBLIC WORKS A SYMPOSIUM

Discussion

By Messrs. Daniel W. Mead, and William J. Wilgus

Daniel W. Mead,[114] Past-President and Hon. M. Am. Soc. C. E. (by letter).[114a]—The writer has read with much interest the discussion that followed the publication of his paper. He feels that little can be gained by continuing a discussion on public works, the facts concerning which, in any exact or detailed way, have been so successfully and completely withheld from the public. The important question is not whether "this" or "that" will be accomplished by any of the various projects which have been thrust upon the American people without their desire or consent, but whether or not a new theory of economic procedure is to be inaugurated in this country and whether business relations shall be changed entirely by the destruction of utility and other business investments through Federal competition. There is, and can be, no doubt that any private business can be ruined and destroyed by Federal competition or by the competition of a municipal or district project which is subsidized by the Federal Government to the extent of almost 50%, and in which taxes can be eliminated and interest reduced to a mere fraction of that which must be paid by private investors. If this is the basis of determining the fair cost of a project or the basis of a so-called "yardstick," then there can be no doubt of the inability of the private corporations to compete. In all such public investments the cost of production is an unknown quantity which can be so manipulated that losses can be covered by taxes, and the "yardsticks" so limited that they will assure ruin to all those the Government (Federal, State, or municipal) would destroy.

The Administration can, if it will, force this country toward Communism, Socialism, Fascism, or any other such "ism" as its unscrupulous politicians

Note.—This Symposium was presented at the meeting of the Engineering-Economics and Finance Division, Boston, Mass., October 7, 1937, and published in February, 1938, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: March, 1938, by J. K. Finch, M. Am. Soc. C. E.; May, 1938, by Messrs. Elliott J. Dent, C. Frank Allen, Bradley G. Seitz, Alfred Allen Stuart, Pierce P. Furber, R. F. Bessey, Donald M. Baker, and Philip W. Henry; June, 1938, by Messrs. H. K. Barrows, Harry A. Wiersema, J. D. Galloway, E. S. Martin, and K. Bert Hirashima; and September, 1938, by Messrs. Louis E. Ayres, John H. Meursinge, and Samuel B. Folk.

[114] Prof. Emeritus, Hydr. and San. Eng., Univ. of Wisconsin; Cons. Engr., Madison, Wis.

[114a] Received by the Secretary February 14, 1939.

may desire, under the plea of emergency or the excuse of furnishing work to the unemployed. This can be done as long as it can maintain its authority and power by subsidizing selfish individuals and communities who prefer personal profit to the public good and are influenced through public works paid for by tax money and public loans imposed upon the taxpayers on a basis that will not stand sound economic analysis.

The writer stands for a fair deal. He has no defense to offer for private dishonesty in either public utilities or private business; neither can he satisfy his ideals of honesty and integrity by approving unfair and dishonest procedure in Federal or other Government affairs because some private businesses may also have been unfair and dishonest. What is needed at this time is intelligence, honesty, and decency in both business and Government.

The writer does not believe that the country can spend itself out of this depression; neither does he believe that business and confidence can be restored and the depression vanquished by Government competition and the destruction of the investments of its citizens.

WILLIAM J. WILGUS,[115] HON. M. AM. SOC. C. E. (by letter).[115a]—A careful reading of the discussions of the Symposium has failed to change the views of the writer that the billion dollar quartet of public works, comprising the Passamaquoddy Tidal Power scheme, the Grand Coulee project, the Fort Peck Dam, and the Florida Ship Canal, are economically unjustifiable. In the case of the Florida Ship Canal the writer finds in the discussions not one single word in its defense. On the record it stands without merit and is unworthy of the further expenditure of the savings of the people of the United States.

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[115] Cons. Engr., Ascutney, Vt.

[115a] Received by the Secretary December 23, 1938.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

DEOXYGENATION AND REOXYGENATION

Discussion

BY C. J. VELZ, ASSOC. M. AM. SOC. C. E.

C. J. VELZ,[18] ASSOC. M. AM. SOC. C. E. (by letter).[18a]—A technique for the evaluation of stream assets and pollution liabilities, in relation to remedial programs, is essential to a sound and economical solution of the pollution problem. Professor Phelps has drawn an apt parallel with highway construction and has shown that the functions and constants in deoxygenation and reoxygenation are established, and that they are comparable in accuracy and applicability to those used by engineers in other fields. When he is designing for specific loads and conditions, in order to be reasonably certain that the structure will fulfill, safely and economically, the functions intended, the bridge engineer subjects his design to a rigorous analysis. Although the sanitary engineer deals with constantly changing, living, forces rather than inanimate, static, forces, knowledge concerning the rates of change are sufficiently established to permit evaluations similar to those made by the bridge engineer. If sanitary engineering is to be placed on a plane equal to that of other branches of engineering, arbitrary selection in design, based on practical experience alone, must become secondary to a systematic technique of analysis based upon a knowledge of the fundamentals involved.

In discussing the rate of stream turnover, Professor Phelps mentions the work of the British Royal Commission[17] in which it is estimated that the rate of mixing ranges from 1 hr to 6 hr. Analysis of the experimental work of the Stream Pollution Laboratory of the United States Public Health Service and comparisons of computed and observed dissolved oxygen profiles of streams indicate shorter periods—$\frac{1}{8}$ hr to $\frac{1}{2}$ hr. Experimental data that are suitable in the analysis of rates of mix are meager, and it is hoped that further flume studies on reoxygenation, similar to those undertaken by the United States Public Health Service (particularly with greater depths and with more definite

NOTE.—The paper by C. J. Velz, Assoc. M. Am. Soc. C. E., was presented at the meeting of the Sanitary Engineering Division, New York, N. Y., January 21, 1937, and published in April, 1938, *Proceedings*. Discussion has appeared in *Proceedings*, as follows: June, 1938, by Earle B. Phelps, Esq.

[18] Prof., San. Eng. and Head, Civ. Eng. Dept., Manhattan Coll., New York, N. Y.

[18a] Received by the Secretary March 2, 1939.

[17] Royal Comm. on Sewage Disposal (Great Britain), 8th Rept., Vol. 1, p. 10, 1912.

control of mix intervals) will bring forth more exact knowledge concerning this factor. Confirmation of the shorter mix period is evidenced from a comparison of computed and observed conditions on the Ohio and the Hudson rivers. H. R. Crohurst[19] has presented rates of reoxygenation, in pounds of oxygen per thousand square feet of water surface per day between sampling stations. They are based upon the assumption that the difference in oxygen balance between April and other months represents reoxygenation that occurred during those months. In a strict sense, such a method of determining rates of reoxygenation is not exact since all discrepancies and errors are "lumped" in the residual oxygen balance and, therefore, are reflected in the rates determined from them. In order to minimize any discrepancies thus included, a stretch of river, and months of the year that show a decided regeneration in dissolved oxygen, were selected by the writer for comparison with rates obtained from the Standard Reoxygenation Curve (Fig. 3).

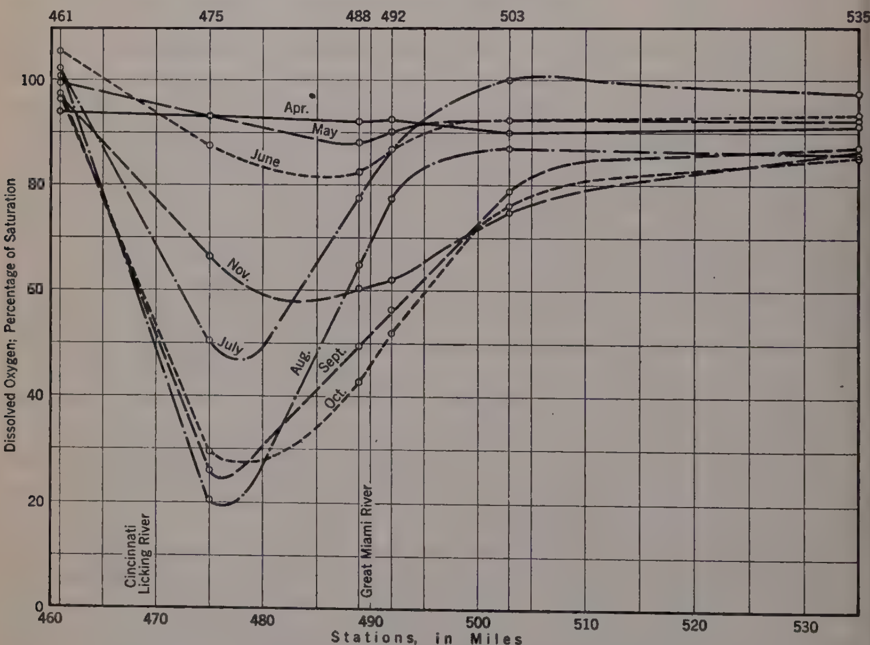

FIG. 8.—DISSOLVED OXYGEN PROFILE; OHIO RIVER, 1930

The dissolved-oxygen profiles of the Ohio River (see Fig. 8) show[19] that, during the months of July, August, September, and October, 1930, the stretch of the river below Cincinnati, Ohio, and between the junctions of the Licking and the Great Miami rivers was subject to a very pronounced regeneration of oxygen content. Utilizing the velocities of flow reported for these months and the corresponding mean rates of stream turnover determined by the author from the experimental data reported by the U. S. Public Health Service,[20] the

[19] "A Study of the Pollution and Natural Purification of the Ohio River," by H. R. Crohurst, *Public Health Bulletin No. 204*.

[20] *Sewage Works Journal*, March, 1936.

writer has computed rates of reoxygenation in this stretch of the river by means of the Standard Reoxygenation Curve. These computed values agree closely with the rates based upon observed dissolved oxygen at Stations 475 and 488 as shown in Table 5. Under Item 4, Table 5, the computations are: Pounds per 1 000 sq ft times 4.9, divided by depth of river, in meters, times mean

TABLE 5.—COMPARISON OF COMPUTED AND OBSERVED RATES OF REOXYGENATION; OHIO RIVER

Item No.	Description	July (1)	August (2)	September (3)	October (4)
1	Velocity, in feet per minute	18.5	11.7	14.1	10.0
2	Stream turnover; time in minutes between mixes, corresponding to velocities in Item 1 (mean of both curves in Fig. 5)	5	22	14	25
	Reoxygenation (Percentage of Saturation per Day; the Maximum Rate with the River at 0% of Saturation):				
3	Computed from Fig. 3; depth = 16.1 ft and temperature = mean of stations 475 and 488*	30.0	18.5	17.9	8.7
4	Observed*; pounds per 1 000 sq ft per day, converted to percentage of saturation value absorbed per day, with the river at 0% of saturation	38.8	19.6	17.4	8.4

* "A Study of the Pollution and Natural Purification of the Ohio River," by H. R. Crohurst, U. S. Public Health Service, *Public Health Bulletin No. 204*.

observed oxygen-saturation deficit, expressed as a decimal ratio. The variation in the rates are within the probable errors of observation and under these conditions the experimental rates of mix are identical with the actual rates of turnover in the river; namely, between 5 min and 25 min.

A further verification of the shorter period of stream turnover is indicated by a comparison of the computed and observed dissolved oxygen values on the Hudson River. In these computations the writer used rates of mix ranging from $\frac{1}{8}$ hr for shallow reaches to $\frac{1}{2}$ hr for deep reaches. The agreement between the observed and the computed profiles is shown in Fig. 9.

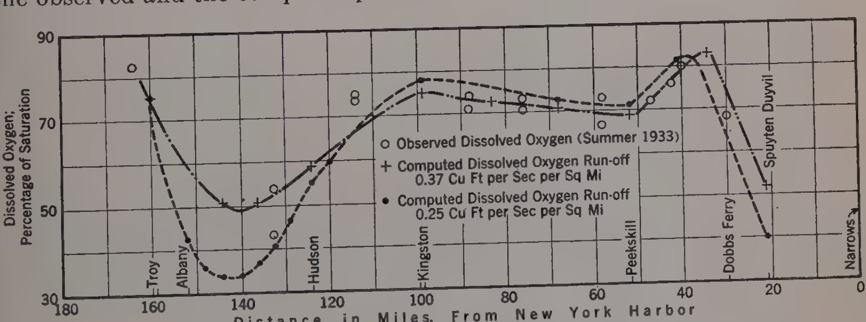

FIG. 9.—DISSOLVED OXYGEN PROFILE, HUDSON RIVER

In addition to verification of the shorter period of stream turnover, the Hudson River data emphasize, strikingly, the importance of the element of depth in reoxygenation.

In the upper reaches from Albany, N. Y., to Hudson, N. Y., the river is relatively shallow and here both observed and computed values of reoxygena-

tion are shown to be pronounced. In passing through this stretch, the river satisfies 90% of a heavy pollution debt and at the same time rebuilds, by reoxygenation, to about 75% of saturation. From Kingston, N. Y., to Peekskill, N. Y., the river passes through a deep gorge section (60 ft to 100 ft); and, with only 10% residual pollution load from up stream, and very small additional pollution loads added in the stretch, reoxygenation through the deep section is unable to satisfy the relatively light demand evidenced by the change from positive to negative slope of the dissolved oxygen profile. Upon entering the shallow reaches of Tappan Zee below Peekskill, reoxygenation is again pronounced, as indicated by a decided increase in computed, and also observed, dissolved oxygen. The sudden drop in oxygen below the Zee is caused by the heavy concentration of pollution from the Metropolitan area.

Quite apart from the verification of the rate of turnover, the important fact to be observed from the Hudson River data is the dominating influence that depth plays in the reoxygenation phenomenon. The only rational approach to computation of reoxygenation which directly employs this important factor of depth is that laid down by Professor Phelps in 1908 in his reoxygenation equation.[3] This formula is the basis for the writer's Standard Reoxygenation Curve (Fig. 3). The importance of depth is frequently obscured by other factors, which has led some workers, particularly Europeans, to express rates of reoxygenation in terms of oxygen absorbed per unit of surface area without designation of specific depth. Such values are applicable only to streams of the same depth; use of these surface rates at other depths may lead to error.

Technique is no substitute for judgment; in the last analysis judgment is the deciding factor in any given situation. It is hoped, however, that the method of analyzing stream assets and pollution liabilities, offered in this paper, will be a useful tool, and will be an aid to sounder judgment in the solution of pollution problems.

The writer wishes to thank Prof. Gordon M. Fair and H. W. Streeter, Members, Am. Soc. C. E., and Prof. Edward W. Moore, for their valuable criticism. In particular he wishes, respectfully, to acknowledge indebtedness to Professor Phelps, whose earlier writings were primarily the foundation for this paper.

Corrections for *Transactions:* In the denominator of Equation (3) delete "4"; on page 771, Lines 2 and 6 of Example 1, change "5 ft" to "10 ft"; in Scale 4, Fig. 3, multiply all graduations by 2; and, on page 773, Line 2, change "5 ft" to "10 ft." In Fig. 5, the relationship between velocity of flow and the logarithm of the frequency of mix remains linear for velocities between 2 and 25 ft per min, but the time between mixes is increased slightly. A revised Fig. 5 will be published in *Transactions*.

[3] "Location of Sewer Outlets and Discharge of Sewage into New York Harbor," Rept. by the late Maj.-Gen. William Black, U. S. Army (*Retired*), and Prof. Phelps, 1910.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

MOTOR TRANSPORTATION—A FORWARD VIEW A SYMPOSIUM

Discussion

By W. W. Crosby, M. Am. Soc. C. E.

W. W. Crosby,[40] M. Am. Soc. C. E. (by letter).[40a]—The conclusion of Mr. Kettering's "salmon" story is good, and his remarks on "where to get the money," should be regarded as fundamental. Although the writer could applaud most, if not all, the remainder of Mr. Kettering's remarks, it is from the ideas just mentioned that he wishes to propose an hypothesis which may account for many of the defects in the highways of the United States as well as suggest an answer to an engineering question that has been widely discussed for years.

Before so doing let the writer note that he has carefully read the paper by Mr. MacDonald, which seems to express, fairly, the attitude of the U. S. Bureau of Public Roads for the past fifteen years. A significant quotation (regardless of the question of fact) in this case is "Highway builders [in the United States] are proceeding on the principle that the utilization of the highways must produce directly the revenues with which to finance their construction" (see heading "Super-Highways").

The third paper of the Symposium, although limited to a narrower field, seems commendably broad in conception. Mr. Smith remarks several times on the fundamental importance of adequate rights of way, which the writer has been emphasizing for years. Mr. MacDonald barely suggests anything on this point although its prompt recognition is even more important in his work outside the cities. In some of its recent publications[41] the writer believes that the Bureau of Public Roads has admitted the need for "elbow room"[42] in the future.

Note.—This Symposium was presented at the meeting of the Highway Division, Detroit, Mich., July 21, 1937, and published in June, 1938, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: September, 1938, by Messrs. F. Lavis, Edgar Dow Gilman, George Hartley, Robert Kingery, R. L. Morrison, and Roy F. Bessey; October, 1938, by Bruce D. Greenshields, Assoc. M. Am. Soc. C. E.; and January, 1939, by Messrs. Robert B. Brooks, and H. George Altvater.

[40] Cons. Engr., Coronado, Calif.

[40a] Received by the Secretary February 27, 1939.

[41] *Roads and Streets*, February, 1939, p. 53; also *Public Roads*, January, 1939, p. 209.

[42] *Proceedings*, Am. Soc. C. E., May, 1938, p. 973.

The writer believes with Mr. Kettering that "Sometimes, people calculate entirely too much in dollars," particularly in setting up ideals—that is, in planning. Often it is necessary for the material approach to ideals and standards to be gradual as funds may be found; but to establish standards on the basis of funds in sight only results in regrets.

This brings the writer to the hypothesis mentioned:

May it not be that an answer to the questions discussed for so many years—"What is the matter with engineering?" and "Why does it not rank with the learned professions?"—lies in the fact that too many of its more prominent representatives cling to the definition of an engineer as one who "can make one dollar do the work of two," and are not imbued with the spirit of one familiar to the Society: "An engineer is one who adapts the forces of nature (including Human Nature) to the benefit of mankind," or perhaps the latest one:[43]

> "An Engineer is one who, through application of his knowledge of mathematics, the physical and biological sciences and economics and with aid, further, from results obtained through observation, experience, scientific discovery and invention, so utilizes the materials and directs the forces of nature that they are made to operate to the benefit of society. An Engineer differs from the technologist in that he must concern himself with the organizational, economic, and managerial aspects as well as the technical aspects of his work."

[43] *Scientific Monthly*, February, 1939, p. 176.

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DISCUSSIONS

BEAM CONSTANTS FOR CONTINUOUS TRUSSES AND BEAMS

Discussion

BY LEO LEGENS, ESQ.

LEO LEGENS,[15] ESQ. (by letter).[15a]—The author shows quite well the limitations of the method of redundant reactions as applied to continuous girders, and also the much greater simplicity of calculation in using the moments over the piers as the redundants. However, he does not show any relationship to the calculation of influence lines by Müller-Breslau's equations.[16] The writer would like to show the application of these equations to the three-span continuous beam of Fig. 2. The moment of inertia of this beam is constant for all spans, which simplifies the calculations as compared to a girder of variable section or truss.

The Müller-Breslau equation[16] states that:

$$X_a \, \delta_{aa} + X_b \, \delta_{ab} \cdots = \Sigma \, P_m \, \delta_{ma} \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

The statically indeterminate parameters X_a, X_b, $\cdots$ are to be chosen so that the deflection curves may be obtained easily and so that the deflections $\delta_{aa}\delta_{ab}\cdots$ will vanish if possible.

Moment diagrams $M_a \, M_b \cdots$ produced by $X_a = -1$, $X_b = -1 \cdots$ should be of simple forms (rectangles, triangles, or parabolas) for which Müller-Breslau[16] has indicated a rapid method of finding the deflections δ_{ma}, δ_{mb}, $\cdots$ with the values ω_R, ω_D, ω_P and ω_P''. These deflections are:

For Fig. 11:

$$E \, I \, y = \frac{z \, l^2}{2} \, \omega_R \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12a)$$

in which $\omega_R = \frac{x}{l} - \frac{x^2}{l^2}$; and, for Fig. 12:

$$E \, I \, y = \frac{z \, l^2}{6} \, \omega_D \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12b)$$

NOTE.—This paper by George L. Epps, Jun. Am. Soc. C. E., was published in October, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1939, by C. W. Deans, Esq.; and March, 1939, by Messrs. George W. Lamb, and A. A. Eremin.

[15] Engr., S. N. C. F. (French Railroads), Strasbourg, France.

[15a] Received by the Secretary December 2, 1938.

[16] "Graphische Statik der Baukonstruktionen," by Müller-Breslau, Band II, Abteilung, 1908 Edition, p. 104 *et seq.*

in which $\omega_D = \frac{x}{l} - \frac{x^3}{l^3}$; and, for Fig. 13:

$$\delta_{ii} = \int M_i^2\, dx = \frac{l}{3}\left(y_1^2 + y_1\, y_2 + y_2^2\right) \ldots\ldots\ldots\ldots (12c)$$

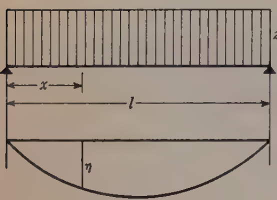

FIG. 11

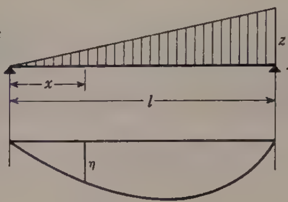

FIG. 12

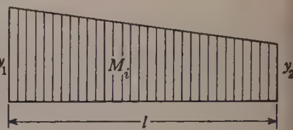

FIG. 13

In this problem only one indeterminate parameter, $X_a = R_1$, is introduced and the beam is treated as a two-span continuous girder (see Fig. 14(a)).

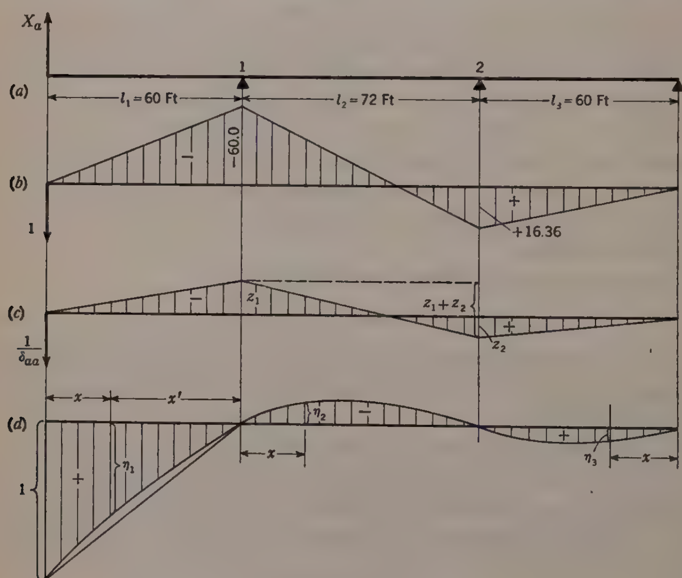

FIG. 14

Parameter $X_a = -1$ produces bending moments M_{1a} and M_{a2} at Supports 1 and 2 which may be calculated by the equation of three moments of Clapeyron:

$$M_{1a}\, l_2 + 2\, M_{2a}\,(l_2 + l_3) = 0 \ldots\ldots\ldots\ldots (13a)$$

and

$$M_{1a} = -1 \times l_1 = -60 \ldots\ldots\ldots\ldots (13b)$$

Therefore, $M_{2a} = \frac{l_1\, l_2}{2\,(l_2 + l_3)} = \frac{60 \times 72}{2 \times 132} = 16.36.$

The parameter X_a is equal to $\frac{\delta_{ma}}{\delta_{aa}}$ for a load of $P_{m=1}$. Since the deflection curve for a load $X_a = -1$ is the δ_{ma} line, the deflection curve for a load $\frac{1}{\delta_{aa}}$ is the influence line for $X_a = R_1$.

According to Fig. 13: $E\,I\,\delta_{aa} = \int M_a^2\,dx = \frac{60}{3} \times 60^2 + \frac{72}{3}(60^2 - 60 \times 16.36 + 16.36^2) + \frac{60}{3} \times 16.36^2 = 146\,300$; $z_1 = -\frac{60}{146\,300} = -0.000140$; and, $z_2 = +\frac{16.36}{146\,300} = +0.000111$. Finally the ordinates of the X_a-line (see Fig. 14(d)) according to Figs. 11 and 12 are:

$$y_1 = \frac{x_1}{l_1} - \omega_D \times 0.000410\,\frac{l_1^2}{6} = \frac{x_1}{l_1} - 0.246\,\omega_D \qquad (14a)$$

$$y_2 = \omega_R \times 0.000410\,\frac{l_2^2}{2} + \omega_D\,(0.000410 + 0.000111)\,\frac{l_2^2}{6}$$
$$= -1.06\,\omega_R + 0.45\,\omega_D \qquad (14b)$$

and

$$y_3 = \omega_D \times 0.000111\,\frac{l_3^2}{6} = +0.066\,\omega_D \qquad (14c)$$

The values for ω_R and ω_D may be written rapidly, even without the aid of a table:

For $\frac{x}{l} = 0.2$

$\omega_R = 0.2 - 0.2^2 = 0.16$; and, $\omega_D = 0.2 - 0.2^3 = 0.192$

for $\frac{x}{l} = 0.5$

$\omega_R = 0.5 - 0.5^2 = 0.25$; and, $\omega_D = 0.5 - 0.5^3 = 0.375$

and for $\frac{x}{l} = 0.8$

$\omega_R = 0.8 - 0.8^2 = 0.16$; and, $\omega_D = 0.8 - 0.8^3 = 0.288.$

These values substituted in the equations for y_1, y_2 and y_3 give the same values for the influence line for R_1 as those shown in Fig. 2. This is a very simple and quick method of computing the influence line for a three-span continuous girder as it allows the use of a slide-rule for all computations.

For a multiple-span girder the writer recommends (as does the author) using the bending moments over the supports as redundants. Thus one can apply the equations of three moments, and the deflection curves are computed for each span separately with the values of ω_D, assuming a constant value of I.

Müller-Breslau[16] presents a large number of examples demonstrating simple methods of computing such systems.

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DISCUSSIONS

ANALYSIS OF RUN-OFF CHARACTERISTICS

Discussion

By Messrs. Franklin F. Snyder, and W. G. Hoyt

Franklin F. Snyder,[22] Jun. Am. Soc. C. E. (by letter).[22a]—A "revision of current methods for the determination of stream flow from rainfall" is presented in this paper. The author uses time of concentration (defined as length of time until run-off becomes constant from a continuous rain of constant intensity) and a basic hydrograph to define the run-off characteristics of drainage basins.

The writer will show that if the user of the unit hydrograph and the user of the basic hydrograph could agree on the time of concentration for any area, the latter would be identical with the former as obtained by use of a unit hydrograph under the same assumptions. Of course, this would require that the agreed time of concentration be equal to the duration of surface run-off from a short duration or instantaneous storm, and would be a different quantity from that used in the rational method of sewer design.

Definitions not given by the author are required, with the introduction of the term, "surface run-off" or "flood run-off," as distinguished from "ground-water run-off." "Surface run-off" is usually defined as storm water that reaches the open drainage channels without penetrating the ground surface; but, the separation of surface run-off from ground-water run-off is usually made on hydrographs by drawing a horizontal or slightly inclined base flow line under the flood rise. The base flow rate is usually fixed by a low value of the stream flow previous to the flood rise. The time involved for the duration of surface run-off varies approximately from three to ten days for areas of from 10 to 10 000 sq miles. The flood run-off thus obtained may, or may not, fit the definition.

Table 9 gives a two-hour unit hydrograph derived by synthetic procedure[16] for the stream flow station of the U. S. Geological Survey at Big Piney Run

Note.—This paper by Otto H. Meyer, Assoc. M. Am. Soc. C. E., was published in November, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1939, by Messrs. Victor H. Cochrane, and Bertram S. Barnes; and March, 1939, by Messrs. LeRoy K. Sherman, Richmond T. Zoch, and Merrill Bernard.

[22] Hydr. Engr., Dept. of Forests and Waters, Commonwealth of Pennsylvania, Harrisburg, Pa.

[22a] Received by the Secretary February 20, 1939.

[16] "Synthetic Unit Graphs," by Franklin F. Snyder, *Transactions*, Am. Geophysical Union, 1938, Part I.

near Salisbury, Pa. (drainage area equals 24.5 sq miles). The ordinates as tabulated are for the end of the designated two-hour periods. Fig. 11 shows the two-hour unit hydrograph and the cumulative plotting of the two-hour ordinates of the unit hydrograph, which is the run-off graph computed with

TABLE 9.—UNIT HYDROGRAPH ORDINATES, IN CUBIC FEET PER SECOND, AT TWO-HOUR INTERVALS FOR BIG PINEY RUN NEAR SALISBURY, PA.

Period	Unit Hydrographs		Period	Unit Hydrographs		Period	Unit Hydrographs		Period	Unit Hydrographs	
	Two-hour	Four-hour		Two-hour	Four-hour		Two-hour	Four-hour		Two-hour	Four-hour
(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
1	120	60	13	150	156	25	54	57	37	12	13
2	480	300	14	138	144	26	49	51	38	10	11
3	1 380	930	15	128	133	27	44	46	39	8	9
4	1 480*	1 430	16	119	123	28	40	42	40	6	7
5	890	1 185	17	110	114	29	36	38	41	4	5
6	530	710	18	102	106	30	32	34	42	3	3
7	360	445	19	94	98	31	28	30	43	2	2
8	280	320	20	87	90	32	25	26	44	1	1
9	230	255	21	80	83	33	22	23	45	....	1
10	196†	213	22	72	76	34	19	20	46	....	....
11	176	186	23	66	69	35	16	17	47	....	....
12	162	169	24	60	63	36	14	15	....	....	....
Totals	....	....	....	....	....	....	....	....	....	7 915	7 909

* 1 650 at seventh hour.
† Recession coefficient of 0.92 applicable after twentieth hour.

the two-hour unit hydrograph for a continuous rain of 1 in. per 2 hr (100% run-off). This, then, is the basic hydrograph for the area according to the unit hydrograph procedure. To show that Fig. 11 is independent of the unit of time, Table 9 also gives the four-hour unit hydrograph for the same area. This hydrograph is the two-item progressive average of the two-hour unit hydrograph. The six-hour unit hydrograph would be the three-item progressive average of the two-hour unit hydrograph, etc.

It should be emphasized that if an area is such that satisfactory results can be obtained with a unit hydrograph for four-hour rainfall quantities, the four-hour unit hydrograph ordinates would usually be tabulated at intervals of four hours rather than two hours as in Table 9. However, there is no reason why two-hour ordinates cannot be used with rainfall tabulated every four hours.

The continuous rain of 1 in. per 2 hr (in this case 2 in. per 4 hr) can be distributed with the four-hour unit hydrograph by the usual procedure of tabulating the product of the unit rainfall, times the unit hydrograph ordinates in vertical columns and adding horizontally for the totals. The hydrograph obtained is exactly the same as the basic hydrograph defined with the two-hour unit hydrograph. The same result would be obtained from a six-hour unit hydrograph and a continuous rain of 3 in. per 6 hr.

The time of concentration by the foregoing procedure is 88 hr, although the last two days do not increase the discharge very much. The basic hydro-

graph obtained from the two-hour unit hydrograph (run-off graph of a two-hour storm) and the author's method of synthesis is shown in Fig. 11. The author's values of λ, given in Table 1 of the paper, were used with ordinates of the two-hour unit hydrograph at 1-hr intervals. The procedure failed to define the concentration curve above a value of 6 245 cu ft per sec, so what was considered a reasonable extension was made and is shown by a broken line in Fig. 11.

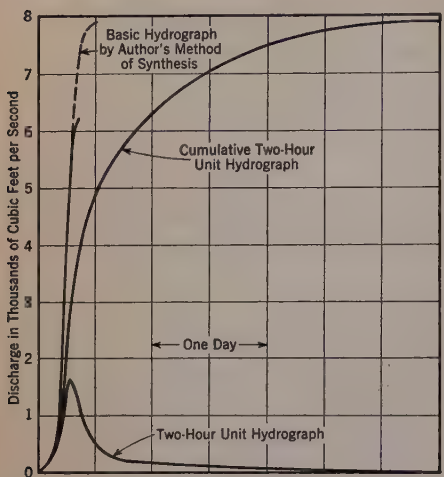

FIG. 11.—BASIC HYDROGRAPHS FOR BIG PINEY RUN NEAR SALISBURY, PA.

This comparison shows the considerable discrepancy between the two procedures due to the fact that the author's time of concentration is evidently much less than the duration of unit hydrographs as ordinarily used.

If the run-off (which is usually called "flood run-off") were true surface run-off as previously defined, the time of concentration of 12 hr, and the basic hydrograph as obtained with the author's procedure, would probably be a true picture of what occurs. However, much of the run-off which at present is called surface run-off is not true surface run-off as previously defined; it is sub-surface storm flow which penetrated the surface of the ground on its way to the drainage channels, but did not reach the stable ground-water table. The recession sides of unit hydrographs are storage depletions, but not from channels alone. In other words, part of the run-off from every unit of rainfall is destined to pass through this temporary ground storage and reach the gaging station later than would satisfy a reasonable value of time of concentration as used in the rational method.

Eventually it may be necessary to separate the true surface run-off from sub-surface storm flow and ground-water discharge since it appears probable that there is a limit to the sub-surface storm-water storage and flow for any particular drainage basin. Thus, in very large floods, due to the maximum being reached or a limit set by existing infiltration capacity, a greater percentage of the total flood run-off occurs as true surface run-off as compared to sub-surface storm flow. This gives a unit hydrograph with an earlier and higher peak and less volume in the recession side than is the case in ordinary floods. This difficulty requires special adjustments in the unit hydrograph procedure and would probably require similar adjustments in the use of a basic hydrograph.

Fig. 12, showing the rainfall and run-off records for the storm of October 27 and 28, 1937, on Big Piney Run, illustrates the foregoing discussion.

A rational time of concentration for the small head-water area, whose longest channel is eight miles with a drop of 300 ft, would not be more than

four hours at the maximum. The lag for the area, which is believed to be a more important hydrologic characteristic of a natural drainage basin than time of concentration, is about six hours. Lag is defined as time from center of mass of surface run-off producing rain to peak discharge and tends to be a constant for any drainage basin.

The U. S. Geological Survey topographical maps show some swamp area bordering on the two main channels, but in general the topography is quite rough, with a range in elevation from 2 800 to 2 200 ft. It is felt that the

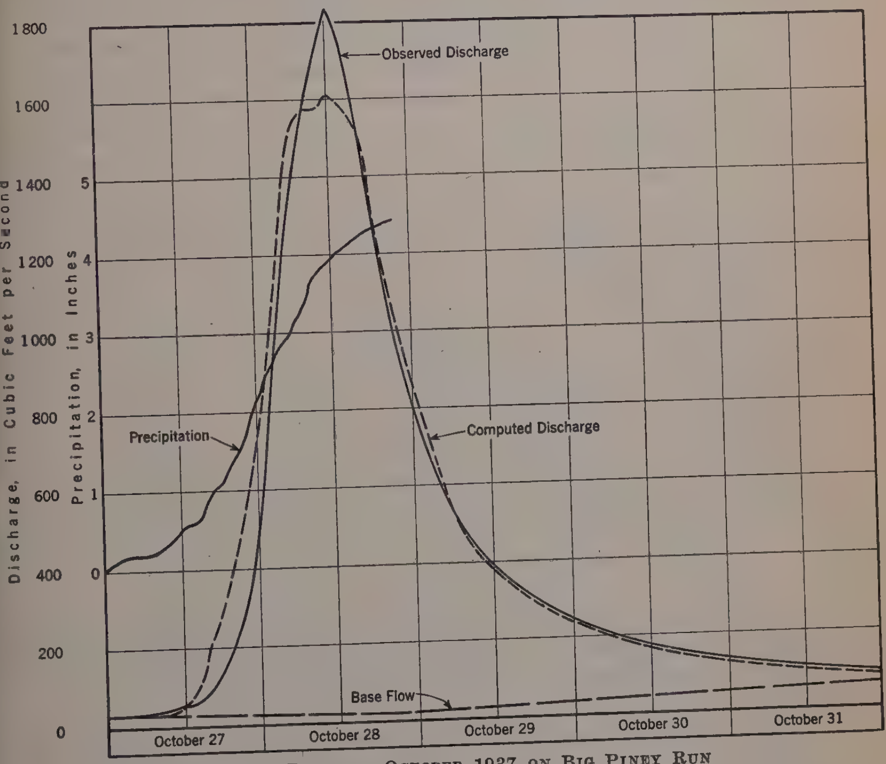

FIG. 12.—FLOOD OF OCTOBER 1937 ON BIG PINEY RUN

swamp areas might be the cause of as much as one hour out of the observed 6-hr lag for the area.

The Big Piney recording rain and snow gage, located on the Big Piney Run Basin, shows a continuous rain of 18 hr of fairly constant intensity, preceded by several short periods of precipitation and followed by 12 hr of precipitation at a lower but fairly constant rate; yet the discharge hydrograph shows no tendency toward a constant discharge at the crest. As a matter of fact, the unit hydrograph procedure for this storm gives a flatter crest than the observed record due to the rainfall of decreasing intensity at the end of the storm.

Fig. 12 shows the discharge hydrograph computed from the basin rainfall with the two-hour unit hydrograph previously given. Table 10 shows the

two-hour quantities of precipitation and surface run-off and the distributions for a few of the items. The computations were shortened through the use of a surface run-off recession coefficient and the addition of a composite column which includes the recessions of discharge of all 2-hr items that have passed the twentieth hour. The increasing percentage of run-off was based on rainfall-run-off relations used in forecasting work in which ground-water discharge is used as the index of run-off conditions.

TABLE 10.—UNIT HYDROGRAPH COMPUTATIONS FOR STORM OF OCTOBER 29, 1937, ON BIG PINEY RUN NEAR SALISBURY, PA.

Time ending, October:	Precipitation, in inches	Surface run-off, in inches	Flow Characteristics, in Cubic Feet per Second							
			Distribution of surface run-off†				Composite term	Surface run-off	Base flow	Computed total discharge
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
27; 2:00 A.M.	0.10*	....	....	....	....	....	....	....	31	31
4:00 A.M.	0.08*	....	....	....	....	....	....	....	31	31
6:00 A.M.	0.02*	....	....	....	...	....	....	....	31	31
8:00 A.M.	0.02*	....	....	....	....	....	....	....	30	30
10:00 A.M.	0.11	0.02	2	....	....	....	....	2	30	32
12:00 Noon	0.18	0.07	10	8	....	....	....	18	29	47
2:00 P.M.	0.08	0.03	28	34	4	....	....	66	29	95
4:00 P.M.	0.28	0.15	30	96	14	22	....	162	28	190
6:00 P.M.	0.20	0.11	18	104	41	72	....	248	28	276
8:00 P.M.	0.27	0.16	11	62	44	207	....	396	27	423
10:00 P.M.	0.28	0.19	7	37	27	221	....	544	27	571
12:00 Mdnt.	0.50	0.37	6	25	16	133	....	699	26	725
28; 2:00 A.M.	0.40	0.31	5	19	11	80	....	927	26	953
4:00 A.M.	0.33	0.26	4	16	8	54	....	1 254	25	1 279
6:00 A.M.	0.28	0.24	....	14	7	42	4	1 491	25	1 516
8:00 A.M.	0.35	0.30	36	....	6	35	17	1 546	24	1 570
10:00 A.M.	0.30	0.24	144	29	....	29	21	1 549	24	1 573
USE OF RECESSION COEFFICIENTS: (21 + 29) TIMES 0.92 EQUALS 46										
12:00 Noon	0.15	0.13	414	115	16	....	46	1 587	23	1 610
2:00 P.M.	0.16	0.14	443	331	62	17	62	1 550	22	1 572
4:00 P.M.	0.13	0.11	266	356	180	67	86	1 388	22	1 420
6:00 P.M.	0.08	0.07	159	214	192	193	113	1 234	21	1 255
8:00 P.M.	0.07	0.06	108	127	116	207	170	1 109	21	1 130
10:00 P.M.	0.05	0.04	84	87	69	125	212	976	20	996
12:00 Mdnt.	....	....	69	67	47	74	242	850	21	871
29; 2:00 A.M.	....	....	59	55	36	50	266	730	22	752
4:00 A.M.	....	....	....	47	30	39	299	604	24	628
6:00 A.M.	....	....	....	....	25	32	318	499	25	524
8:00 A.M.	....	....	....	....	....	27	315	429	27	456
10:00 A.M.	....	....	....	....	....	....	315	384	28	412
12:00 Noon	....	....	....	....	....	....	310	349	30	379
2:00 P.M.	....	....	....	....	....	....	298	319	31	350
4:00 P.M.	....	....	....	....	....	....	285	293	33	326
6:00 P.M.	....	....	....	....	....	....	270	270	34	304
Totals........	4.42	3.00	....	....	....	...	....	....	....	....

* Initial loss.

† The distributions are omitted for the items of run-off from 4 P.M., October 27 to 6 A.M., October 28, and for the items after 2 P.M., October 28.

It would appear from a study of this storm that any basic hydrograph that would reproduce the discharge hydrograph would have to be quite similar to that obtainable from the unit hydrograph.

Assuming that a basic hydrograph can be agreed upon for any area, there are several weaknesses in its analysis and synthesis as given by the author.

If a time of concentration of 12 hr is assumed for the drainage basin whose basic hydrograph is developed in Fig. 2, it is seen that the time of peak discharge is earlier for the 7-hr rain than for the 1-hr rain. The possibility of such an occurrence seems very slight.

Another point is the fact that if rain ceases after some unit area, say the sixth, has contributed its h-value at the gage, the author's procedure immediately begins to decrease the ordinate of discharge contributed at the gage by this area which is six units of time away from the gage. This means that the five units of flow from the area which have already started their trip through the drainage system are decreased simply because it stops raining.

The author's use of a recession coefficient is very timely. This coefficient eliminates about two-thirds of the work involved in unit hydrograph computations.

The author's illustration of the fact that ground-water discharge is a reliable index of run-off conditions is to be commended. However, the statement that ground-water flow has no characteristics to identify it with any particular period of precipitation is not so acceptable. If ground-water discharge is to be used as a successful index of run-off conditions, a well-defined procedure must be available for delineating ground-water discharge. It is difficult to accept time of the year as a satisfactory medium for this purpose. Ground-water depletion and storage curves provide a ready means of identifying, quantitatively, the ground-water discharge or recharge with particular periods of precipitation.

The statement that the unit hydrograph procedure and a continuous rain will frequently give a run-off rate exceeding the product of the rainfall rate and the percentage of run-off is difficult to understand. The continuous rain of 1 in. per 2 hr and the two-hour unit hydrograph for Big Piney Run give a peak discharge (100% run-off) of 7 915 cu ft per sec which is, of course, a run-off rate equal to the rainfall rate of 0.5 in. per hr.

In considering the probable accuracy of the author's synthesis of a hydrograph as compared with the unit hydrograph procedure, especially in flood forecasting, it is doubtful whether the proposed methods are far superior to the unit hydrograph procedure. It has been the writer's experience that the three main problems in computing an accurate discharge hydrograph from rainfall are the rainfall itself, the quantity of run-off that will occur, and the shaping up of the run-off.

Of the three, the possible rainfall eccentricities provide the most trouble, and the shaping up of the discharge hydrograph is the easiest to accomplish. By this, it is not meant to infer that the assumption of a uniformly distributed rain of constant intensity might not be the best for certain flood control studies. In comparing the flood reduction benefits available at some particular point from reservoirs at various locations, it may be more desirable to have a consistent procedure rather than to duplicate probable quantitative occurrences.

W. G. Hoyt,[23] M. Am. Soc. C. E. (by letter).[23a]—Through detailed analyses, such as described by the author, a mass of valuable information is gradually

[23] Hydr. Engr. (Prin.), Conservation Branch, U. S. Geological Survey, Washington, D. C.

[23a] Received by the Secretary March 8, 1939.

being accumulated relating to the characteristics of surface run-off resulting from precipitation in the form of rain. It appears that at present the application of the unit graph and similar methods, such as that described by the author, must be largely confined to problems involving flood run-off resulting from rain, and to areas where the run-off reaches stream channels essentially as over-land flow. In the eastern part of the United States its application to flood problems during periods of no snow has been demonstrated amply. Unsatisfactory results may be obtained, however, if application is attempted in areas or periods when the flood run-off results largely from melting snow, or in areas where the greater part of the stream discharge reaches channels through the ground. Under such conditions failure to substantiate earlier findings may be attributed erroneously to flaws in basic principles, whereas the difficulties may relate to a lack of knowledge regarding the behavior of run-off that may reach a stream through the ground with a responsiveness approaching that of direct surface run-off.

Most of the basic data underlying the theory of the unit graph, the basic hydrograph described by the author and, in fact, much of the present knowledge relating to the flood run-off have been based on observations made in the north-central and northeastern parts of the United States. In these regions, relatively: (*a*) The greater part of the flood flow is essentially direct surface run-off; (*b*) infiltration rates and retention capacities over large areas, during major flood periods, are relatively small; and (*c*) periods involving large quantities of melting snow are generally found inapplicable for study. It is only natural that some modification in principles thus derived may be necessary before they are applicable in parts of the central and much of the western part of the United States. In this latter region, flood run-off—especially from the Sierra Nevada, Rocky, and northern coast range mountains—is largely associated with run-off from melting snow; and, in the southern coast range and semi-desert regions, infiltration rates and retention capacities, in general, are apparently very high, and only a small part of the storm precipitation reaches stream channels as direct over-land flow.

It is in the latter areas, especially, that much more hydrologic research with reference to flood run-off is urgently needed.

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DISCUSSIONS

SIPHONS AS WATER-LEVEL REGULATORS

Discussion

By T. J. Corwin, Jr., M. Am. Soc. C. E.

T. J. Corwin, Jr.,[15] M. Am. Soc. C. E. (by letter).[15a]—The use of siphons for the control of water level, or for spillways, is of particular interest to the hydraulic engineer since a close regulation of water surface is possible. Mr. Stevens has presented a paper that represents a most thorough study on the action of siphons.

The design, arranged to have both a low head or high head on the siphon, is most unusual and can be used to advantage where enough head is available to make this layout possible. The adjustable primer can be used where unusual operating conditions will not permit a constant maximum water surface elevation.

Siphon spillways were selected to discharge excess flows occurring in the Tiger Creek conduit of the Mokelumne River power development of the Pacific Gas and Electric Company, in California, on the basis of reliability, close regulation of water surface, low cost, and freedom from maintenance. The conduit with a capacity of 550 cu ft per sec, completed in 1931, is about 20 miles long, of which 16 miles is concrete flume 14 ft wide and 7 ft deep. At several points along the conduit, feeder flumes or pipe lines pick up the flow of tributary streams. During certain seasons, the run-off from heavy rain or melting snow increases the flow from these feeder flumes to such an extent that the main flume would be overtopped unless protected by automatic spillways. Seven siphons were installed at points where feeders enter the conduit line, and tests were made on four, to check performance and to effect possible improvements in the design.

Fred C. Scobey, M. Am. Soc. C. E., A. W. Kidder, Assoc. M. Am. Soc. C. E., and the writer, made these tests in April, 1932.[16]

In so far as they bear upon the paper by Mr. Stevens the following conclusions from this test may be cited:

Note.—This paper by J. C. Stevens, M. Am. Soc. C. E., was published in October, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1939, by Messrs. A. Griffin, and H. P. Currin and D. M. Umphrey.

[15] Asst. Engr., Pacific Gas & Elec. Co., San Francisco, Calif.

[15a] Received by the Secretary December 13, 1938.

[16] "Results of Tests on Siphon Spillways for Large Flumes," by A. W. Kidder and T. J. Corwin, *Engineering News-Record*, May 5, 1932, p. 649.

(1) The lower end of an inclined barrel should be either sealed or should have some vertical bends to cause the water to jet across the barrel;

(2) An auxiliary priming weir seems justified where the rise in water surface is limited (the depth required for priming was reduced about 2 in. when the auxiliary device was operating);

(3) Use of the lowest throat height consistent with good proportions, for the barrel, tends to decrease the head required for priming;

(4) The vacuum breaker pipes could be placed directly in front of the inlet rather than at the side since there is no appreciable surface disturbance when an enlarged inlet section is used;

(5) The air vent, which has an area of about 6% of the throat area, should be placed about 2 in. above the desired water surface elevation at which the discharge should stop (the reason for this is that the siphon continues to operate at a partial discharge, sucking air and water through the vent until the water depth in the flume is correct for the setting of the throat of the siphon); and

(6) The use of more piezometer connections should be made for a complete study and the oscillation of the mercury columns should be damped. The rate of rise of the water surface should be controlled.

The operation of these siphons was free from objectionable vibration at all times, although the addition of sealing pools had been blamed for pulsations of some siphons. The pool had no apparent effect upon the discharge capacity.

Any designer contemplating the location of a siphon spillway in a conduit or small forebay must be sure that the water is free from entrapped air. The writer has had experience with a siphon located in a structure with two sources of water. It would prime with the water diverted from the stream but would fail to function when the excess water entered the structure in a turbulent manner from a pipe line.

Without a doubt, Mr. Stevens' test is the most thorough ever made and published. The energy line is of particular interest and shows where the major losses occur in the barrel, and the designer might be able to improve the coefficient of discharge by reducing the radius of curvation of the bends in the barrel.

It is interesting to note that the heads required for priming are about the same for the Tiger Creek and Walterville systems, and that the rate of rise in the water surface was found to influence the priming head in both cases. The apparent installation of trash-rack bars in front of the siphon shown in the Walterville structure might reduce the effectiveness of the siphon unless automatic rakes are provided to keep the racks clean.

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DISCUSSIONS

SPECIFICATION AND DESIGN OF STEEL GUSSET-PLATES

Discussion

BY RUSSELL C. BRINKER, JUN. AM. SOC. C. E.

RUSSELL C. BRINKER,[19] JUN. AM. SOC. C. E. (by letter).[19a]—Technical literature on the design of steel gusset-plates is much too scarce, and therefore Mr. Rust's paper is a welcome one. Rule-of-thumb design has doubtless been practised more on this "detail" than on any other unit in modern steel bridges. Nevertheless, safe designs (although perhaps often uneconomical) have resulted, and the following statement in the "Synopsis" might well be challenged: "Since a gusset-plate is a point of discontinuity of stress transfer in a structural steel frame, it is here that failures most often occur." Unquestionably gusset-plates are points of discontinuity and, therefore, are danger spots, but search of technical literature will disclose few if any instances of major (or minor) failures caused by faulty gusset-plate design. Perhaps that is merely an indication that rule-of-thumb design has been so overly cautious that even in otherwise faulty designs the gusset-plates have been too generously proportioned to permit failure, but it does prompt the statement that, practically, gusset-plates do not fail.

It might be noted that in general, gusset-plate design is based upon two conditions: (1) Selecting a size of plate such that the bearing value of a rivet upon the plate will approximately equal the double shearing value; and (2) providing sufficient area along any section of the gusset-plate to take the stress upon that section (brought by the rivets). As for condition (2), for specifications allowing 24 000 lb per sq in. in bearing and 12 000 lb per sq in. for shear, an $\frac{11}{16}$-in. plate will give balanced rivet values. For most modern spans, gusset-plates are usually made $\frac{5}{8}$ in. thick, or slightly less than the thickness required for balanced rivet values.

A point that has been neglected—apparently without serious results—is the possibility of buckling of the plate. In rare cases, stiffening angles or

NOTE.—This paper by T. H. Rust, Assoc. M. Am. Soc. C. E., was published in November, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1939, by Messrs. R. H. Sherlock, and L. E. Grinter.

[19] Asst. Prof. of Eng., Univ. of Hawaii, Honolulu, Hawaii.

[19a] Received by the Secretary January 13, 1939.

"flats" may be riveted along the edges of the plate to prevent the occurrence of such failures. The fact that a reduction in thickness requires more rivets, with a consequent enlargement of the plate, has tended to keep the plates heavy enough to prevent failure.

An important series of investigations performed to determine desirable sizes and shapes of gusset plates has been discussed by Theophil Wyss.[20] Two points worthy of note in his results were: (1) The maximum angle over which the stress spreads from the member through the gusset-plate is about 30°; and (2) an analysis can be made of the condition by which gusset-plates receive direct stress and also a twisting moment, in the same manner that a beam is designed for direct and bending stress by means of the formula,

$$f = \frac{P}{A} \pm \frac{Mc}{I} \quad \text{.........................(5)}$$

[20] "Forschungsarbeiten Auf Dem Gebietes Des Ingenieurwesens," by Theophil Wyss.

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DISCUSSIONS

STATE-WIDE SURVEYING PRACTICE IN MASSACHUSETTS

A SYMPOSIUM

Discussion

BY ELLIOTT B. ROBERTS, M. AM. SOC. C. E.

ELLIOTT B. ROBERTS,[20] M. AM. SOC. C. E. (by letter).[20a]—Added emphasis may well be given one aspect of the subject treated by Mr. Houdlette. Reference is made to the difficulties experienced in the past by those who have tried to popularize the use of geodetic control for the correlation of local surveys; namely (1) inadequate distribution of geodetic control points, and (2) a widespread reluctance of surveyors to undertake the somewhat specialized computation necessary in adapting geographic position data.

Mr. Houdlette's organization has gone far to remove the first of these difficulties in Massachusetts, and a few more years of such work may well see the State so well supplied with control points that few, if any, surveyors could conscientiously avoid controlling their work by use of the national geodetic datum. The U. S. Coast and Geodetic Survey, in designing adequate plane co-ordinate projections for the States, and publishing tables relating thereto, has largely resolved the second difficulty. When that Bureau completes its intended publication of the plane co-ordinates of all its triangulation stations, it will have done so completely. Naturally the well-known operations of so diligent an organization as the Massachusetts Geodetic Survey are a tremendous added force in popularizing the plane co-ordinate projection in Massachusetts.

The emphasis intended to be given by this discussion is to the fact advanced by Mr. Houdlette that, with abundant local control points defined in plane co-ordinates, the local surveyor, to use them, has no specialized computations to make, no new surveying process to learn, and nothing unusual to do but to

NOTE.—This Symposium was presented at the meeting of the Surveying and Mapping Division, Boston, Mass., October 7, 1937, and published in November, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1939, by Messrs. H. J. Shea, and Philip Kissam; and March, 1939, by Messrs. Lloyd G. Frost, William Bowie, and R. M. Wilson.

[20] Hydrographic and Geodetic Engr., U. S. Coast and Geodetic Survey, Washington, D. C.

[20a] Received by the Secretary February 2, 1939.

start from the nearest control points instead of at random. From there on he may make the usual survey, computing latitudes and departures in conventional manner, and adjusting for the closure errors as he has always done. He may make a perfectly traditional plane survey; yet there is the tremendously important difference that his work is on a standard grid rather than an arbitrary local one. His work is based on the national datum, and is therefore correlated with every other such controlled survey throughout the entire country. This simple innovation, therefore, can eliminate the multiplicity of origins, mostly unrelated, the over-supply of height datums, and the economic loss of thousands of uncontrolled surveys of only momentary value.

The foregoing is so simple that it may seem a trivial point. The writer has observed, however, that many engineers are preoccupied with technicalities and fail to "pound home" the simple fundamentals necessary to impress others. The simple fundamental here is that by accepting a standard origin and coordinate system instead of setting up local ones of their own, surveyors can reap great benefits. This truth should be shouted, figuratively, from the housetops, so that surveyors may be persuaded henceforth to work in harmony instead of at cross purposes.

Mr. Houdlette is to be congratulated for the untiring productive work he has done, and for his excellent presentation of the subject. It is to be hoped that similar programs may be adopted in all the States. Massachusetts is learning that planning pays, in surveying as well as otherwise, and all the States should become aware of the great unnecessary cost attending the present haphazard methods.

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DISCUSSIONS

TRAFFIC PROBLEMS IN METROPOLITAN AREAS

Discussion

By Messrs. D. Grant Mickle, Robert A. Mitchell, and Cleveland B. Coe

D. Grant Mickle,[6] Assoc. M. Am. Soc. C. E. (by letter).[6a]—In his excellent paper Mr. Reeder shows that city streets are not filled with human and mechanical disaster as is often thought. With the opportunities for conflict within the various traffic streams and eddies existing in metropolitan areas, it is indeed surprising that there are not more deaths and injuries than there are.

It is the problem of traffic engineers to schedule the movements of vehicles, persons, and merchandise, within the traffic stream, so that they are coordinated and made orderly and safe. Among the factors to be considered in such a program are the arrangement of the circulatory system, land usage, location and density of population, location of employment in offices, retail establishments and industries, and the location and usage of recreational and educational centers. The relation of these factors to one another, and their influence upon congestion and accidents, make up the traffic problem.

Because accidents constitute the spectacular phase of the traffic engineer's work, there has been, and still is, an insistent public demand that direct and positive action be taken to stop them. Mr. Reeder has presented many of the methods used in this direct approach. Automatic traffic-control signals, stop signs, speed control zones, and stated speed "slow" signs are among the control methods described. In connection with the latter device, it should be emphasized that it is difficult to enforce a sign reading "Slow to 11 Miles," inasmuch as the officer must pace the offender for a reasonable distance in order to obtain a conviction for an offense. Furthermore, care must be taken that a local ordinance provides authority for such a control device.

It is indeed true that the pedestrian problem is one of increasing seriousness. Many city officials, recognizing this problem, are providing legislation which places responsibility upon the pedestrian, as well as upon the motorist, in his

Note.—This paper by Earl J. Reeder, Esq., was published in December, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1939, by Messrs. George H. Herrold, and Thomas Buckley.

[6] Mgr., Traffic and Transport Dept., Jensen, Bowen & Farrell, Ann Arbor, Mich., and Asst. Director and Planning Engr., Michigan Highway Planning Survey.

[6a] Received by the Secretary January 18, 1939.

activities in the traffic stream. No longer does the pedestrian have uncontested right to cross the street whenever and wherever he pleases. His actions and behavior are prescribed by law, and the designers of traffic-control signals are beginning to include indications in the signal to allot certain parts of the cycle to the pedestrian, restricting his movement during the remainder of the cycle.

One of the most prevalent locations for pedestrian fatalities is at the very outskirts of cities along the major highways. Sidewalks are often lacking at such places, highway lighting may be poor or non-existent, and the speed of vehicles is generally high. These factors combine to make the zone at the fringe of cities produce fatalities at a high rate.

In Michigan it was found in 1938 that, of all fatal pedestrian accidents on rural trunk-line highways, 34% occurred within the zone extending 1 mile out from the city limits. The rate per mile was found to be more than twice that of the second mile zone, decreasing gradually from that point as the distance from the city increased. This study focuses attention upon a pedestrian problem in which both the rural and urban officials must co-operate.

The genuine causes of accidents and congestion in metropolitan areas are usually of such a fundamental nature that their correction is costly. For example, in Detroit, Mich.,[7] it was found that the city's pattern of streets radiating from the central business district emphasized the conditions of congestion and hazard caused by the present-day use of the automobile in cities planned for other kinds of transportation. Studies of traffic volumes, and of the origin and destination of traffic, disclosed that these down-town streets carried not only vehicles bound for destinations within the district, but also thousands of other cars which passed through them to distant destinations because adequate cross-town arteries were lacking. Investigation of parking conditions, requirements, and habits indicated that desirably located off-street facilities were inadequate and that all of the legal, and much of the illegal, curb space was fully occupied. Not only were terminal facilities insufficient but the radial arterial streets were over-crowded for several miles away from the central business district.

The results of these unfavorable conditions upon business were revealed when comparisons of traffic counts over a period of twelve years showed that the number of people entering the district had actually declined although passenger car volumes had increased. Recommendations for dealing with these conditions included the construction of new cross-town arteries and the substitution of ample storage space in the areas occupied by deteriorated property surrounding the business district. In making these recommendations, the engineers went well beyond the act of merely counting and analyzing traffic movements. They took into consideration both the general physical plan and organization of the city, and the social and economic implications of real estate conditions in and about the central business district.

Too often officials and the public fail to realize the magnitude of the traffic problem or the losses attendant on congestion and accidents. The work of the traffic engineer, therefore, is often handicapped through lack of sufficient

[7] "Street Traffic—Detroit," Michigan State Highway Dept., 1936–37.

funds or proper interest in his plans. For example, a traffic report in Toledo, Ohio,[8] revealed that traffic accident losses amounted to $1 022 325 annually, as compared with annual fire losses of less than one-third of that sum, or $311 802; yet more than $2 was allotted to the fire department for each dollar of actual fire loss, whereas the traffic control and betterment services received only 34 cents for each dollar of traffic accident loss. Similarly in Louisville, Ky., whereas an average of 89 cents was spent to prevent one dollar's worth of fire loss, only 30 cents was spent to prevent an equal amount of traffic loss.

Parking or storage facilities in the business districts of cities are far from adequate; and providing these facilities is a problem of major importance. It is thought by some authorities that the provision of terminal facilities in the business district is a public duty, and if this is true, means for public ownership of parking facilities must be devised. Another view is that the primary use of the public streets is for moving traffic, and the parking of vehicles is a secondary consideration which should not interfere, to any appreciable degree, with the free movement of vehicular or pedestrian traffic.

Confusion is apparent in the attitude of the business men, who tend to demand strict regulation of all curb space, except that adjoining their own property. This conflict cannot be reconciled entirely; but it can be equalized by giving consideration to all the related factors, among which are traffic volume, vehicular speed, available space, the demand for spaces, and parking-time requirements.

The main function of time-limit parking regulations is to provide reasonable turnover and equable distribution of available curb space. Some cities are turning to parking meters as a means of enforcing time-limit regulations.

Business men realize that parking facilities are essential to business and that costly decentralization will result unless the problem is solved. Officials are attempting various means of improving off-street parking facilities or means of improving mass transportation facilities in an effort to curb decentralization tendencies.

A traffic survey in Youngstown, Ohio,[9] compared the number of persons accumulated in the business district by the use of mass transportation facilities with the number that used passenger automobiles. The average number of persons transported per mass carrier was 35.72, whereas the average per passenger automobile was 1.48. There can be no question of the relative efficiency of these two types of carriers in regard to economy in the use of street space. If the number of persons using mass transportation facilities had to resort to passenger automobiles as a means of transportation, and if the average of 1.48 persons per passenger automobile remained the same, 29 730 additional passenger vehicles would be required. The streets could not possibly accommodate this additional load. Thus, it will be seen that mass transportation is a vital element in the traffic problem of Youngstown, as it is in any large city; and that, in any plan of traffic control and regulation, its

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[8] "Traffic Planning Report—Toledo, Ohio," Jensen, Bowen & Farrell, Engrs., 1937.

[9] "Traffic Planning Report—Youngstown, Ohio," Jensen, Bowen & Farrell, Engrs., 1937–38.

requirements must receive due consideration, as must the effect of its operation on the free movement of traffic.

Mr. Reeder has shown the need for a three-fold attack upon the traffic problem involving the use of engineering, enforcement, and education. A balanced traffic improvement program must include all of these methods of approach.

In conclusion, the first consideration that any city should give to its traffic problem is the efficient use of its present facilities. Minor physical improvements often may be made to the methods of control or to the street system which result in making the present facilities more useful and more efficient. In planning for future facilities, the basic facts concerning traffic movement should be studied carefully and taken into consideration in developing a traffic-planning program. If traffic control and enforcement are to be successful, all users of street space must be made aware of their responsibilities, one to another, and educated to the safe and efficient use of these facilities.

ROBERT A. MITCHELL,[10] ESQ. (by letter).[10a]—In any discussion of this paper one must keep in mind the enormous scope of such a subject and realize that it is impossible to cover even the most important problems fully in such a small space. This is understandable when one realizes that entire books have been written on the subject.

Considering the limitations, Mr. Reeder has presented a sufficiently clear picture so that those who peruse it can obtain an adequate idea of some metropolitan traffic problems, their bearing on social and economic life, and how such conditions may be alleviated. On the other hand, the writer questions the accuracy of certain statements; for instance (see "Helping Traffic Through Danger Spots"), Mr. Reeder infers there is a widespread belief that a city street is "a continuous scene of human and mechanical disaster." On the contrary, the safety problem is so difficult to correct because very few people have seen, or been involved in, a traffic accident, and thus do not take sufficient interest in the subject. It is also unfortunate that more factual data and examples were not used to substantiate certain assertions, because civil engineers have analytical minds and are impressed by facts and figures.

Under the heading "Helping Traffic Through Danger Spots" the author presents, admirably, the advantages of careful engineering planning in making streets safer by designing, locating, and operating traffic signals, signs, and markings so as to take the "guess-work" out of driving. A person could probably learn to run a locomotive as easily as he could a motor vehicle, but no one would think of letting him run that locomotive even a mile along the track unless he knew all the rules of the road and had carefully planned signals to guide him.

The importance that Mr. Reeder attaches to accident records as basic functional data for correcting "bad spots" cannot be stressed too greatly. However, some who have had practical experience in handling traffic problems in a large city find that there are many "mix-ups" that must be solved by

[10] Traffic Engr., Dept. of Public Safety, Div. of Traffic Eng., Philadelphia, Pa.

[10a] Received by the Secretary January 17, 1939.

past experience and personal judgment. These are mostly problems of congestion and delay rather than those of hazard.

Careful engineering planning also plays an important part in decreasing delays which, Mr. Reeder states (see "Synopsis"), defeat, in part, the purpose of street traffic. From a practical standpoint, citizens seem just as much interested in having traffic officials "clean up" points of serious congestion as accident "sore spots." The author's discussion of by-passing through traffic, parking, and loading and unloading conditions yields an insight into some of the factors causing congestion and indicates practical methods of correcting the situation; but, there are other phases worthy of consideration.

A well-designed traffic signal system, for instance, accelerates traffic, reduces delays, and eliminates congestion, in addition to simplifying the traffic movement, and reducing accidents. When Broad Street in Philadelphia, Pa., was controlled by police at each intersection, the usual sight was a solid mass of vehicles for ten or fifteen continuous blocks—a condition breeding congestion and accidents. Under a proper signal system, over-all speed was increased, the volume of traffic increased, and accidents decreased. Vehicles now travel in small successive groups, often called "waves" or "platoons." In addition, the timing can be altered each day so as to favor the heavy inbound traffic in the morning and the peak outbound traffic in the afternoon. On one of the other down-town streets, the traction company's daily records show that the delay time of their trolley cars has been reduced 18% by the operation of a progressive signal system.

A little white or yellow paint applied in the right places can also help in reducing delays. Unless three lanes of traffic are definitely marked off on each half of Broad Street, motorists will only drive in two lanes, thus reducing the capacity one-third. There are also many examples of increasing the capacity of intersections to handle traffic by the installation of proper laning. Sometimes certain types of cement, rubber, or metal markers are used instead of paint.

Congestion on streets carrying a large volume of traffic can often be alleviated by creating one-way streets or establishing other convenient adjacent routes. Two parallel and adjacent streets in Philadelphia, both with 44-ft cartways, were two-way for many years and carried considerable arterial traffic. In spite of some public criticism, these streets were made one-way in opposite directions and to-day (1939) each of them is carrying about 80% more traffic with less congestion and delay. Traffic on another important artery was relieved by smooth-paving an adjacent granite block street with asphalt, and building, at small cost, a few connecting streets.

Safety zones at street-car stops allow motorists to move by the trolley while it is stopped for passengers; channelization islands at certain intersections reduce conflicts and speed up traffic movement; at highly congested points where land values are not too high, traffic circles and grade separators are sometimes used to improve conditions and there are several examples of parkways and elevated highways in New York, N. Y., Chicago, Ill., Newark, N. J., Philadelphia, and other cities, where millions of dollars have been spent to relieve traffic congestion.

Mr. Reeder made no truer statement than that: "The pedestrian problem is one of increasing seriousness" (see "Protecting the Person on Foot"). While the number of car occupants killed in traffic accidents in large cities has been steadily decreasing, the proportion of pedestrian fatalities has increased until to-day (1939) such deaths represent approximately 75% of the total. In the space allotted, the author has covered the subject admirably well particularly the need for selective education.

Paper after paper has been written on the value of adequate street lighting in improving metropolitan traffic problems. Fact after fact has been presented by experts to show the proportionate seriousness of night accidents over day accidents and to prove that the improvement of illumination in certain cities and on certain highways greatly reduced night accidents. Nevertheless, most public officials are doing almost nothing to improve street lighting in metropolitan areas and the writer does not believe they will until money becomes more plentiful, and there is a strong public demand.

Mr. Reeder "puts his finger" on what is probably the greatest difficulty facing public officials in their attempt to correct traffic problems in the business districts of municipalities; that is, "One of the fallacies that must be exposed and overcome is the feeling that is often held by merchants and business people that traffic density is a good sign of business prosperity" (see "Reducing Traffic Concentration"). Counts made in one large city in 1938 show that on the average, a street car carries 30 people, a bus 15, and a private car less than 2 into the central business district; and yet, merchants continually bring pressure to bear on the law-making body to defeat regulations designed to make it easier for mass transportation riders to reach the shopping district.

Motor vehicles of to-day are smooth running machines which move along the road at reasonably high speeds without any apparent effort. This has made speed control a real problem since most motorists fail to realize how fast they are traveling. Mr. Reeder has explained, adequately, some of the methods for coping with the situation, such as speed zoning and signal systems; but neither of the solutions will be effective unless, as he says, they are reasonable, adequately "sold" to the public, and based on engineering investigation. One of the main reasons that speed zoning has not been effective is that unreasonable speeds were established and then the motorists were given to understand they could travel about 10 miles faster before they would be arrested. During the last year of the 40-mile speed limit in Pennsylvania, state authorities even published the fact that motorists would not be apprehended unless they drove faster than 50 miles per hr. In another case, police officials informed the writer that they never apprehended a driver unless he drove faster than 30 miles an hr through a speed zone posted for speeds not greater than 20 miles per hr. How can one expect motorists to obey regulations that the officials themselves do not believe proper? Establish a reasonable speed, enforce it to the letter, and then motorists will respect it, and other laws. Most of the credit for Pennsylvania's reduction of traffic fatalities in 1938 is given to the absolute enforcement of the speed limit of 50 miles per hr adopted in the latter part of 1937.

Herein, the writer has introduced data to show the importance of mass transportation carriers, which substantiate Mr. Reeder's assertions as to "the street space used per person carried" (see "Mass Transportation") by various types of vehicles. With the increased use of private vehicles, the problem of mass transportation is increasing, both financially and from a standpoint of congestion. There has been considerable controversy as to whether or not buses will supplant street cars entirely because they are more mobile and seem to interfere less with the general traffic movement. One thing is certain: Quick pick-up, more speed, and faster stopping ability must be the qualifications of modern transportation vehicles.

Although Mr. Reeder hasn't covered all the problems faced by municipal traffic authorities, the toughest of which is "public relations," he has given enough information to impress the reader with the immensity of the traffic problem and to show that it will not be solved by trial-and-error methods but by adequate traffic planning based on engineering principles.

Cleveland B. Coe,[11] Assoc. M. Am. Soc. C. E. (by letter).[11a]—The point most forcibly "brought home" by this paper is that traffic control is now a specialized branch of engineering, calling for the usual engineering procedure of assembling all available facts, analyzing them, and creating a rational design according to tested rules. The rules will continue to be more or less empirical, as the human element plays a greater part than in the design of structures. Efforts in the past to force traffic, both wheeled and foot, into certain channels have failed to a greater or less degree, and the solution lies in the education of the individual, on the one hand, and in making traffic rules and channels easy and natural to follow, on the other.

It required centuries to educate the public to the employment of engineers in structural design; and in the fields of highway location and construction, and land surveying, as well as others, the process of education is still going on, with results showing in proportion to the density of population of any section of the country. It is a clear function of technical societies to educate the public, as no individual can do so. Neither can headquarters of a national group reach the civic consciousness of the voters of the many cities, with the possible exception of a few of the larger ones, where in all probability experts are already in charge of the problem, driven thereto by the exceeding seriousness of the situation.

This is a matter for local sections and local engineering clubs. Each one should have a committee working through local publicity channels, with subject matter furnished by their own investigations and by central headquarters. They should not confuse their objective with an attempted solution of local traffic problems, but should educate the local authorities to the point of employing a traffic engineer, first laying the groundwork by educating the voting public and the press to approval of this course.

The importance of this groundwork cannot be over-estimated. Time and again in all fields, notably in City Management, the employment of experts has

[11] Asst. Highway Engr., TVA, Chattanooga, Tenn.

[11a] Received by the Secretary January 31, 1939.

been tried and abandoned for just such lack of support, the voters objecting most strenuously to bringing in an outside expert, no matter whether one was available in the community or not. It is just such lack of understanding and intolerance of the democratic process that has caused most engineers to be exceedingly ineffective politicians.

An interesting fact revealed by the data in the paper is the apparent numerical relation between licensed drivers and pedestrian fatalities. The decrease in fatal accidents in 1938 is encouraging. It may be attributed to the widespread campaign of education. No doubt part of it is due to the education of the younger generation in the proper handling of a high-powered dangerous weapon and their general knowledge, when on foot, of the limitations in driver judgment and mechanical equipment. Many writers have commented on the psychological change that occurs in an otherwise courteous and well-behaved individual when he or she slides under the wheel of an automobile, and it is to be hoped that the coming generation will be so gradually accustomed, from childhood, to the control of so much power that no sudden change of habit will occur when the legal driving age is reached.

Education of the public as to the by-passing of through traffic and limiting storage of cars on pavement provided primarily for moving traffic is a more difficult problem and one requiring much time to effect. Cities in the more densely populated Eastern States have apparently learned their lesson through bitter experience, and highways are now being routed around centers of population. Cities in the Southern States are still in the stage where the merchants complain loudly against any proposal to take through tourist traffic off of Main Street. This attitude seems to be not so uncompromising of late, but it is still thought necessary by many that parking be unrestricted in front of their doors. Facts on parking are comparatively easy to ascertain, and only facts will convince the merchant that little or no trade comes to his store from a car occupying valuable space in the street.

It should be noted by those who argue that expensive pavement is laid for moving traffic and not for storage that, regardless of restrictions, the public will "pull over" to the curb and stop, if only to discharge a passenger, and that only one or two, doing this in a block, occupy a lane against moving traffic almost as effectively as a line of parked cars. Since this lane is not available anyway, its use should be divided among the drivers as far as practicable and this means limited parking. The use of parking meters to assist the regulating officers is still on trial, some cities and merchants approving them after trial and others having discontinued their use. Here again personal bias and prejudice play a part far too great, whereas the facts could, and should be obtained by an engineer and a report rendered which should quash, effectually, all opinions based on preconceived notions, always excepting those held by people who cannot be reached by fact and reason. All of this emphasizes the importance of employing a trained engineer who cannot only gather the facts but can analyze them and present a summary and conclusions in a form that any citizen can understand.

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DISCUSSIONS

THE YELLOW RIVER PROBLEM

Discussion

By Messrs. Herbert Chatley, and H. van der Veen

Herbert Chatley,[19] Esq. (by letter).[19a]—This paper has the merit of being the first fairly complete technical description of the river in question, and the authors are to be congratulated on having been able to present so full an account.

The references to the Emperor Yü are a little unfortunate. Modern archeological research tends to show that this person is very largely mythical and an imaginative creation of the early Chou dynasts who wished to show that in overthrowing the Shangs they were recreating the ideals of their remote ancestors. There is no reliable history of China prior to about 1500 B.C.

In the references to modern studies some mention should be made of the work of the engineers of the League of Nations, Messrs. A. T. Coode and W. Nijhoff. It is true that a note as to their recommendations occurs in the paper, but their report[10] as a whole is also a valuable contribution to the rather scanty literature on the subject.

On the matter of dike positions and the Dresden experiments, the writer has published [20] a note in 1938 as to the theory of flow between set-back dikes, showing that, if the slope is constant and the flow formulas in current practice are accepted, the bank-edge position does give a larger flow for certain limited rises than a set-back position for the dikes unless the set-back is very large indeed. This conclusion is strongly and very naturally questioned; but, if it is not true, the flow formulas are somewhat incorrect and the hydraulic radius is no longer the sole criterion of flow in a bed of uniform material. The changes in the velocity-depth relation are possibly such that unless river sections are homologous, or nearly so, the mean velocity is not only an exponential function

Note.—This paper by O. J. Todd, M. Am. Soc. C. E., and S. Eliassen, Assoc. M. Am. Soc. C. E., was published in December, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1939, by Messrs. J. W. Beardsley, and Elliott J. Dent.

[19] Former Engr. in Chf., Whangpoo Conservancy Board, and Consultant to the Chinese Government; Cons. Engr., London, England.

[19a] Received by the Secretary January 27, 1939.

[10] "Hydraulic and Road Questions in China," Series of League of Nations Publications, 1936.

[20] *Engineering*, December 9, 1938.

of the hydraulic radius but depends also on the shape of the section. It is worth remembering that, for constant area and constant wetted perimeter, there are two forms—one very narrow and deep and the other very broad and shallow—and it would be of great interest to know if model experiments really showed that the flow in each (with the same slope and bed material) were the same.

In this connection the writer strongly supports the desire of the authors to have exhaustive model investigations, including a careful study of the distortion effect due to vertical scales differing from horizontal ones. The writer is convinced that one great objection to the bank-edge dike position is the "plugging" of the section after a spate. In a flashy river a large deposit of silt must occur when the river suddenly falls, and this will leave a diminished section when the next rush comes. The old bed of the Yellow River south of Shantung is very interesting; in many places it is quite high above the plain. The writer is satisfied that this is largely explicable by the conditions which existed after the break of 1853. Some water was in it when the break occurred farther up; and, in addition, some water continued to enter it after the break, and the silt content of both forthwith settled and plugged the channel. It is improbable that the bed, say, at the crossing of the Grand Canal by the old channel, was as high when the river flowed in it as it is now. The writer urged this argument to Mr. Freeman when the latter advanced his straight deep channel proposals, the writer stating that after a spate the channel would tend to choke and that a new spate would back up and overflow the dikes before the plug was washed away. In this connection it should be noted from the data in Tables 7, 8, and 9 that the silt flow in a few days may represent a large fraction of the entire annual silt flow.

The gradients given in Table 1 show that the Yellow River is much steeper than the Mississippi River, even as the latter is steeper than the Yangtze. The most comparable rivers appear to be those of Northern India, several of which act in much the same way as the Yellow River. Owing to the far greater rainfall, however, and the shorter distance from the mountains, the bed material there is generally coarser. The hypotheses advanced by Mr. Gerald Lacey[21] as to regime conditions are well worth studying in this connection, although they are modified in cohesive materials.

The statement that the discharge of the Yangtzekiang above Ichang approaches 2 500 000 cu ft per sec is a little questionable (sentence preceding "Hydrological Considerations: Frequency and Magnitude of Floods"). This value almost represents the maximum flow in the lower part of that river, and as many large tributaries enter below Ichang such discharge can rarely if ever occur in the gorges.

It is noticeable that the packing of the consolidated loess is quite close, the Terzaghi pore-factor being less than unity.[21a] In the suggested measures for regulation no mention is made of the plan suggested by Capt. W. F. Tyler[22]

[21] "Uniform Flow in Alluvial Rivers," *Minutes of Proceedings*, Inst. C. E., Vol. 237 (1934), p. 421; also, "Stable Channels in Alluvium," *loc. cit.*, Vol. 229 (1929), p. 259.

[21a] Correction for *Transactions:* On p. 1949, Line 38, denominator, change "Weight of water silt" to read "Weight of water and silt."

[22] "The Conservancy of the Yellow River"—Chinese Maritime Customs Special Reports (about 1905).

for silt deposition basins. In view of the fact that there are areas not far from the river which are below its summer level it would be feasible to draw off considerable volumes of silt and water and thereby build up, between dikes, a dike of enormous width. The technical and administrative difficulties would be quite serious but not quite insuperable.

As to the delta, the writer finds some difficulty in crediting the statement that the earlier surveys have much value for comparative purposes, and the backwater function seems to be over-rated; but this is a matter for investigation.

The breach of 1938 presumably occurred after the authors had submitted their paper. The flow has apparently been transferred almost entirely into the Huai River and unless the dikes are repaired there is a serious prospect that the Hungtze Lake may become silted up and that the valuable rice lands on the east side of the Middle Grand Canal may be injured during 1939. In this connection the plan of threatened areas, given in Fig. 35, should be extended down to the Yangtze River, near the junction with the Grand Canal.

The writer sincerely hopes that American and other engineers will devote some thought and investigation with models and otherwise to the problems of this great river, which is the example par excellence of a silt-bearing stream. Messrs. Todd and Eliassen have provided sufficient data for excellent work to be done in this direction.

H. van der Veen,[23] M. Am. Soc. C. E. (by letter).[23a]—The authors' contribution to the study of the problem in question is undoubtedly the most valuable and most extensive treatise on this subject which thus far has been published and contains a mass of data never collected before. When the writer severed his connections with the Chinese Government in 1929, practically no hydrometric research work had been done as it was impossible to induce the Chinese officials to undertake to sponsor it. Lack of funds in those days, and an insufficient understanding of the importance of such investigations, were no doubt responsible for this attitude. It was extremely gratifying, therefore, when, in 1919, Mr. Freeman began his independent hydrometric study of the Yellow River in connection with the crossing of that river by the Grand Canal.[3]

It may truly be said, therefore, that Mr. Freeman was the one who gave the impetus to a further study of the Huang Ho, which Mr. Eliassen in his capacity of Engineer with the Yellow River Commission in Kaifeng, so ably continued and enlarged upon. At the same time it is to be regretted that the authors (as it seems to appear) paid so little attention to the work of J. G. W. Fijnje van Salverda[24] which was based on the reports of Captain P. G. van Schermbeek and Mr. A. Visser. This memorandum was submitted in 1887 to the Chinese Government; and, apart from describing completely the conditions of the Yellow River at that time, and explaining its peculiar characteristics, it offers

[23] Former Advisor in River Conservancy to Chinese Government; Govt. Engr., Bureau Rijks Water Staat, Nijmegen, Netherlands.

[23a] Received by the Secretary February 9, 1939.

[3] "Flood Problems in China," by the late John R. Freeman, *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1405.

[24] "Memorandum Relative to the Improvement of the Huang Ho or Yellow River in North China," by J. G. W. Fijnje van Salverda; submitted to the Chinese Government in 1887.

valuable suggestions about possible methods for ameliorating existing conditions. More particularly, attention is drawn to the fact that much might be attained by constructing adequate protective devices at the foot of each loess cliff so as to prevent the enormous influx of silt, with which the river must cope. Mr. Freeman prized this memorandum so highly that he photographed it. He told the writer that it inspired him to learn more about that interesting subject. Mr. Freeman's passionate interest was roused by this work. Eminent engineer that he was, he saw his opportunity and grasped it to collect the valuable data which he published later;[3] but apart from influencing Mr. Freeman to shed some more light on the pecularities of the Yellow River, the memorandum contains some valuable advice which, if followed, would have improved conditions to such an extent that much of the harm done since would have been prevented. In fact, the same may be said of every sensible suggestion pertaining to improvements of the Yellow River conservancy which have been made since that time; but, as it is, much has been written about the subject and many theories have been advanced, but no work of any practical constructive value has been done.

Problems have been permitted to "drag along" as in the past thousands of years. Formerly, this was due, perhaps, to the mental attitude of the river authorities in China which is best explained by the following anecdote: Once when a high Chinese official was asked why it was that no one had taken radical steps to end all those dike breaks instead of resorting only to temporary measures, he is said to have given this reply: "When one is keen on tigerhunting and there is only one tiger, would it then be sensible to kill that tiger outright during the first hunt?" There may have been some justice in this viewpoint in the old days, but it certainly does not apply to the present generation with its many brilliant young engineers who desire nothing better than to end the untold miseries which the Yellow River continues to inflict. With laudable zeal and energy these younger men have started by practising what they learned in foreign schools and universities about scientific river engineering, following the example set by foreign experts. They began to collect data after approved methods—valuable data, as is proved by the excellent paper under discussion. They even went so far as to have a model made of the river, at Obernach in Germany, where Prof. H. Engels in 1932 (assisted by Dr. Ing. Futu Li and in 1934 by Dr. Ing. Yi Shen), conducted the necessary investigations to determine whether it would be feasible to cope with the silt evil (1) by narrowing the river between dikes, (2) by building the dikes parallel, or not parallel, to the channel, and (3) by creating a straight or a curved channel either with or without wing dikes. These studies are all quite interesting, but it is questionable whether such scientific studies[25] may not be considered rather premature at this stage because, judging by the facts, the practical sides of the question seem to be receiving too scant attention.

It is quite in order to collect data and to conduct intensive investigations since this is necessary in order to arrive at the best solution. It is even an indispensable condition for ultimate success; but it is quite a different matter

[25] "Modelversuche über das Verhalten eines Schemmstofführenden Flusses, etc," by Dr. Ing. Chr. Keutner, *Die Bautechnick*, February 11, 1936.

(and a serious one) when, in pursuing such a course, the immediate needs seem to be forgotten. By paying too much attention to certain sides of a question appreciation of the main issue is likely to suffer. It is somewhat like seeing a few particular trees but not the forest of which they form a part. Why not focus all available attention on one point only instead of spending years in a laboratory in order to ascertain, for example, which shape of river is best suitable to silt transportation? In this particular case, this is the immediate prevention of dike breaks.

It is true, as the authors state (see heading, "Scope of Investigations: Hydraulic Laboratory Experiments"), "The results gave some startling indications." Wherein lies this startling quality? The authors (and the writer, for that matter) have made a serious study of the Yellow River problem, over a long period of years, and are consequently in a position to arrive at conclusions after mature consideration. Surely, to such students there can be nothing strange about the result of investigations. In the first place the investigations proceeded from premises that are not comparable with those of the Yellow River, and secondly (as the authors remark, see heading "Scope of Investigations: Hydraulic Laboratory Experiments") the conditions bore little relation to this river. Furthermore, it is only logical to assume that, in a heavily silt-laden river, less scour can be effected in the main channel when all the silt must be transported than when a great part of this silt is allowed to settle on wide foreshores. There can be no question as to the fact that the more the river is allowed to spread over a large area like that offered by the foreshores the weaker the current will become and the more silt will be deposited. The latter circumstances, undoubtedly, will tend to place the main channel in a better position to maintain its full capacity, always provided that the banks are stabilized so that it cannot widen itself at the cost of depth, and, consequently, energy. At the same time it should not be forgotten that the process of silting the foreshore is accelerated. The banks will become higher and higher until ultimately they will only become inundated during extraordinarily high stages. Then there will be only one channel in any case. Would it not be better, therefore, to try to find a solution whereby this inundation can be postponed, at least, and, in the meantime, try to evolve methods that would minimize the influx of silt? Researches in this direction are useful indeed, and it is gratifying to note that the authors did considerable in that respect.

Under "Introduction" the authors state: "One of the real obstacles to progress, of course, lies in the lack of hydraulic knowledge of the river itself." Is this entirely true? Would it not have been possible to construct certain works which, at least, would have prevented the annual breaks, notwithstanding that deplorable lack of data, a shortcoming that could always have been remedied during the execution of protective measures? By beginning to pay too much attention to "many of the perplexities that have shrouded the river in mystery" the vision on the problem as a whole may become more or less blurred.

What is the problem in a broad sense? In a report[26] on the Yellow River problem submitted in 1921 to the Ministry of the Interior, the writer dealt with

[26] *De Ingenieur*, Royal Dutch Inst. of Engrs., 1934, No. 33 (abstract).

this subject in detail and also gave advice as to what should be done to mitigate the danger of, if not to prevent, the disastrous dike failures of the Yellow River.

In principle, the Yellow River is a watercourse, pure and simple, draining its catchment-basin and, like other rivers, transporting detritus. Every river fulfills this duty in its own particular way. Thus, depending on the quantity of loose soil which happens to be available on the drainage area, the steepness of its slopes and that of the tributaries, different rivers will bring more or less detritus down to the low lands and eventually, if conditions are favorable, as far as the sea. If the slope is too gentle in the low countries to maintain the energy required to keep the silt load in suspension, this load is dropped. Depending on the quantity being carried, it is either merely deposited along the banks of the river when, during floods, the water overflows (with the result that natural levees are formed with gentle slopes away from the river), or it also settles to a greater or lesser extent in the river channel itself. In the latter case, during an unusually severe freshet, the river will probably break through one of its levees after a certain lapse of time and from then on will follow a new course in which the same cycle is repeated. This is called plainbuilding, a process which continues on and on until eventually a certain equilibrium is reached, whereby the slope of the plain has become sufficiently steep to enable the river to develop a gradient that is capable of imparting to the current the energy that is required to carry all the silt as far as the sea. In the case of most rivers plainbuilding is completed, or practically so, because erosion in the hills has discontinued (or at least has diminished in intensity), or because the topographic features of the country lend themselves to a speedy solution of the problem. Unfortunately, the Yellow River basin has neither exhausted its supply of silt not does its geographical location render a rapid solution possible. Between the hills and the sea there is an enormous plain which the river must traverse. At the same time the supply of silt seems to be unlimited. Consequently it is reasonable to conclude that because the river is passing through the middle phase of its stabilization (a gigantic process) it would be foolish not to abide by it; or, what is still more foolish, to try to fight against it. Only if river engineers are prepared to accept this fact as indisputable, will it be possible to succeed in ameliorating the conditions affecting the Yellow River problem. In this connection it will be well to recognize the fact that such steps will only tend to mitigate an unpleasant state of affairs; they will never be able to solve it except in a degree.

A few thousand years ago, when the Chinese began to build dikes along the Yellow River they interfered with Nature's process of plainbuilding, inasmuch as they restricted the process to a narrow strip of land. The effect, of course, was to hasten the forming of a steeper gradient which, in turn, created a stronger current so that more silt could be transported to the sea. Furthermore, the river was made to run on a ridge, with the result that if Nature now asserts its right to continue plainbuilding the gravest consequences ensue for those standing in her way.

To make a sweeping statement, it would have been infinitely better for the present generation if past generations had permitted the Yellow River to do its work unhindered, or if those in the past who were entrusted with its care had

been far-sighted enough at least to permit the river to follow another course. This was not done, however, and therefore the present generation is faced with the fact that the river runs on a ridge where it is kept in bounds between dikes which are more or less far apart. Within this cramped space Nature goes on with its process of plainbuilding in an endeavor to establish a slope sufficiently steep to create an equilibrium between the volume of silt that must be carried, and the carrying capacity of the current. It is evident, therefore, that, as long as such a slope has not been established, this narrow strip of a plain (which, by Nature's right, belonged entirely to the River) will continue to rise higher until the final stage is reached. The question is, "When will that event occur?" Perhaps this moment might have been accelerated if: (1) The space allotted to this restricted plainbuilding had been restricted still further—in other words, if the dikes had not been so far apart; and (2) if the main channel had been given a fixed alignment. However, for the moment, it is probably best to leave this question alone.

The river channel is left free to shift from one side to the other between the dikes. Time after time, therefore, the dikes are exposed to a direct attack of the current, at one place or another. Inevitably, they collapse and permit the disastrous inundations in the low-lying, adjacent, great plain. The higher the ridge on which the river runs becomes, the more devastating will be the consequences of a dike failure. Why then, one may well ask, are not better precautions taken to protect those dikes? In the course of many centuries the Chinese have developed clever devices for dike protection; but these are only applied whenever a dike is actually threatened; never before. This seems illogical, but it is often difficult, if not impossible, to predict the particular spot at which the dike will be attacked during a freshet season. It would be necessary in order to afford entire safety from flood to protect every foot of dike. It is clear that such a procedure would be enormously costly; and the Chinese are not a race that throws money away at random. Why, then, protect a dike when one is not sure that it will be necessary? This system is mainly responsible for the bad reputation of the Yellow River, whatever its other bad points may be.

The crux of the question is in dike protection; and it is quite immaterial for the moment, whether dikes should be far apart or not, which dimensions should be given to the channel so as to enable it to carry a maximum quantity of silt, whether highly silt-laden water follows the same hydraulic laws as clear water, etc. All these are extremely interesting problems in themselves and, therefore, should receive due attention in time; but at this stage they are of no consequence.

Since it is beyond question that the main cause of dike failures is the direct attack of the river (that is, the deep-water channel) on the dike, it is obvious that the only method of preventing those attacks is to prevent the main channel from running too close to the dikes.

In the writer's report[26] to the Chinese Government he explained in detail how this could be done. The method he advocated was to construct an adequate system of groins and parallel works with a view to keeping the channel in a fixed position. As a matter of fact the authors advise, in principle, a similar procedure. The width that should be given to the channel is a matter of conjecture, at least for the present. It would be advisable, however, to make it ample at the outset because it is far less costly to lengthen the groins,

in case they should prove to be too short, than to make them shorter. Over a certain length these regulation works should not be higher than the foreshore, although they should slope upward slightly from the new channel alignment (normal lines) in the direction of the dikes. Perhaps this will render it necessary, at some places, to give them a slight elevation with respect to the general ground level. The remaining length of the groins—that is, the distance outside of the future high-water channel—can be raised above high-water level.

Groins should be built close together in those stretches where the new river alignment is designed close to the present side of the channel. Of course, it is not necessary to extend each groin as far as the main dike; but a series of groins may be grouped together by means of a dam parallel with the river, connecting the latter with only one dam across the foreshore with the dike. It is the writer's firm belief that in this way the danger of breaks can be reduced to a minimum. At the same time the channel, being kept within bounds, cannot widen itself during freshets as it does now; consequently no dissipation of energy can occur (at least from that cause) so that, incidentally, more energy will be available for carrying silt. Whatever the relation may be between the velocity of the current and its capacity for keeping silt in suspension, there can be no doubt that a river which is free to erode its channel (or if capable of maintaining one course, widens it at the expense of serious scour) cannot possibly have the same capacity for carrying a load as a channel whose width is fixed.

After these structures have been built, or rather during their construction, the hydrology of the river should be studied rigorously. The writer is a fervent adherent of hydraulic studies and has a firm belief in the usefulness of laboratory experiments; but, at the same time, he is of the opinion that great care should be taken to insure that the practical aspects of the problem are not pushed into the background. Consideration of practical aspects should always predominate.

Under the heading "Flood Control and Regulation: Channel Stabilization Methods," the authors describe a procedure that coincides practically with that of the writer.[26] The latter differs only in comparatively minor points such as the elevation of the groins, and their direction. In Holland where groins have been used extensively there are adherents to each mode of location, different directions being advocated, with different supporting arguments in each case. There is something attractive in deflecting groins, however, as they tend, somehow, to concentrate the current; but, after all, their radius of action is small compared with the width of the river, so that it is doubtful whether they are worth worrying about. That laboratory experiments in this connection "are likely to repay themselves a hundred-fold," therefore, seems to be a rather strong statement.

There is one other item on which the writer would like to comment; namely, the question of spacing the groins. More than a century of experience in Holland has taught engineers that the spacing should preferably be not more than one and a half times their length, and even less if that length should be disproportionate to the width of the channel. Hydraulic laboratory experiments conducted in 1938 supported this view. The spacing has little to do with "a more or less erosion-resisting quality of the soil," however. It is governed principally by the fact that it determines "boundary conditions": The better the spacing, the better these conditions will be, inasmuch as friction

is reduced to a minimum and no energy is wasted at the cost of loss of head and other losses. If that result has been attained, the river banks are adequately protected, whatever the quality of the soil may be. It is a well-known fact that flow can be made smoother by careful planning of training works and, although turbulence plays an important rôle in keeping the heavier particles moving, in this particular case, where the silt is extremely fine, more can be gained by saving energy than by wasting it in order to create turbulence. However, laboratory tests should certainly be made in an effort to shed more light on this question.

Under the heading "Flood Control and Regulation: Influence of the Delta," the authors have verified the writer's views on the same question on the basis of the interesting data which they have collected. In fairness to Mr. van Salverda,[24] however, it should be noted that as early as 1887 he called attention to this particular item. In this connection, it may be of interest to recall the fact that, acting on the advice given in Mr. van Salverda's report, the Chinese Government purchased a bucket dredger in order to clear away the silt in the channels through the delta, just as the authors now propose exactly fifty years later. This dredge arrived in the early nineties; but it foundered soon after it was placed in service and now lies deeply buried in the silt which it was supposed to remove. It would not be surprising if the Chinese in those days ascribed this mishap to a revenge of the river gods who were angered that mortals dared to interfere with their work. The writer does not belong to the class of higher beings who, according to the belief of the masses in China, rule the waters; neither is he angered about what was done; and yet he too does not approve of the method advocated by the authors for the simple reason that it seems to him like "fighting windmills." It seems strange that the authors, who so admirably indicate the means of mitigating the silt evil under the general headings "Hydrological Considerations," and "Flood Control and Regulations," should include dredging through the delta as a remedy. The effect of dredging would be insignificant compared with the excellent measures which they propose in the aforementioned chapters, with which the writer is entirely in accord.

The writer does not believe in detention basins in the hills, however. These must be considered as temporary measures only, without any lasting merit. The writer is keenly in favor of spillway basins in the plain, however, not so much because they will reduce the discharges and flatten the peaks of the freshets, but because they will tend to desilt the water. In doing this the natural plainbuilding process with which Man has interfered is at least restored to some extent. For the same reason the writer considers the by-pass channels mentioned by the authors to be an excellent means of ameliorating conditions.

Before ending his comments on the authors' valuable paper the writer wishes to state that he is fully convinced that if the Chinese Government could be induced to accept the theories so ably expounded by the authors, and could be made to put into practice the remedies that have been advocated, with the least possible delay, the untold miseries, which heretofore have made the Yellow River the scourge of China, would cease to exist.

It seems regrettable that the authors have left China and thus, perhaps, lack the opportunity to continue their work in connection with the fascinating Yellow River problem.

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DISCUSSIONS

ENGINEERING GEOLOGY PROBLEMS AT CONCHAS DAM, NEW MEXICO

Discussion

BY H. L. JOHNSON, ESQ.

H. L. JOHNSON,[8] ESQ. (by letter).[8a]—The geological conditions at Conchas Dam and the major problems resulting from them, are presented in this excellent paper. The author demonstrates the application of geological knowledge to foundation and allied engineering problems. Mr. Crosby states (see heading "Geology of the Dam Site") that, in the upper red shale, the percentage of clay sizes and voids is higher in the south abutment than in the north abutment. This condition was indicated by the initial and incomplete tests run on the shales from both abutments; but a study of the complete data from all tests run on the shales indicates that the average composition and physical characteristics of the red shale in the north and in the south abutments are identical for all practical purposes. The average results for voids and density on a total of 114 samples are presented as Items 1 and 2 in Table 4. The voids in all samples were completely saturated. The average results of grain size determinations on a total of fifty-eight samples, with respect to clay sizes, are given in Item 3, Table 4.

TABLE 4.—CHARACTERISTICS OF RED SHALE

Item No.	Description	South abutment	North abutment
1	Percentage of voids	24.93	24.11
2	Density, in pounds per cubic foot	146.88	147.33
3	Clay sizes (percentages)	37.04	38.05

The author is correct in stating that the red shale is unstratified and that the proportions of grain sizes vary considerably from place to place, as do other characteristics of the shale. It is important to note that this variation is not regular, either horizontally or vertically, but is erratic in all directions. Thus,

NOTE.—This paper by Irving B. Crosby, Affiliate, Am. Soc. C. E., was published in January, 1939, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[8] Conchas Dam, N. Mex.

[8a] Received by the Secretary March 13, 1939.

instead of regular zones, layers, or lenses of material having definite characteristics, the shale consists of irregular interlocked areas of varying characteristics which grade into each other without any definite line of demarcation other than an occasional color break.

The shear and compression values, used and cited by the author as a basis for determining the competency of the red shale, are confined to those obtained from two and three definite horizons in the south and north abutments, respectively. However, values actually used in determining the competency of the shale were representative of the full vertical and horizontal range of the shale at the abutments. In excavating the south and north abutments shale samples were taken, in so far as construction progress and facilities permitted, throughout the red shale exposed at the abutments so that the average characteristics could be determined for the full range of the material in question. In all, fifty-three shear tests and more than two hundred compression tests were made. In connection with the author's statement that, "The average shearing strength as computed from unconfined compression tests on twenty 2-in. cubes was 266 lb per sq in." (see heading "Practical Problems," following Case IV), it should be pointed out that all shear values entering into the design and stability computations were derived from direct shear tests on cylinders of shale 6 in. in diameter.

In the discussion of problems connected with the red shale Mr. Crosby refers to that part of the dam "resting upon the shale" and the parts of the dam "underlain by red shale." It should be made clear that no part of the Main Dam is founded directly on shale and that the entire dam is underlain by shale strata at depths of from 20 ft to 80 ft below the foundation sandstones. The terminal monoliths founded on the canyon sandstone are underlain by both the upper and lower red shales whereas the central monoliths, founded on pink shaly and artesian sandstone, are underlain by only the lower red shale.

That some settlement will occur because of the load of the dam and the presence of the underlying shales has been an accepted fact since the inception of the project. That there may be some differential settlement between those parts of the dam founded on the canyon sandstone and those founded on the artesian sandstone has been recognized and provided for in the design of the abutment monoliths. However, it is doubtful that the settlement differential will approach the 4 in. indicated by the author, whose computation is apparently based on the omission of the lower red shale from consideration or on the assump- that the lower red shale will be equally reconsolidated under the entire dam. This lower red shale has not been uniformly unloaded by erosion, that portion in the canyon being unloaded to a greater degree than those underlying the abutments or canyon walls. Hence, the degree of expansion due to erosional unloading (and, therefore, the resulting degree of reconsolidation possible in the lower red shale under the load imposed by the dam) must be greater in the canyon area than in the area underlying the canyon walls or abutments. Furthermore, the reconsolidation load imposed by the dam on the lower red shale will be much greater under the monoliths between the abutments, 200 to 235 ft high, than under the monoliths, 40 to 95 ft high, on top of the abutments. It is evident, therefore, that although the total thickness of shale underlying

the high monoliths is only slightly more than one-third of the total that underlies the abutments it will unquestionably undergo materially greater reconsolidation. Thus the lower red shale is an important factor because it will tend to act as an equalizer of settlement between the high and low parts of the dam.

In addition to the construction problems involved in excavating the abutment faces in the upper red shale and constructing the abutting monoliths, there was also the problem of insuring a water-tight joint between the upper red shale and the adjacent monoliths. This problem arises from the fact that the shale shrinks appreciably upon exposure to air, with a resultant loss of moisture content. This drying and shrinking produces innumerable hair cracks and causes the opening of the old planes of movement (slickensided fracture planes) which traverse the shale in all directions and frequently are of considerable extent. Every effort was made to prevent drying of the shale by spraying it with a bituminous sealing compound as soon as possible after exposure. However, experience has shown that this treatment is not wholly effective and some drying and shrinkage, with the resultant development of cracks and opening of old fractures, is unavoidable. These cracks, especially those resulting from the opening of old fracture planes, are more or less inter-connected and provide a possible path of leakage around the abutments. In addition to the shale problem there was also the possibility that some differential settlement would occur between the abutments and the abutting monoliths and that shrinkage in the concrete of the abutting monoliths would tend to open the abutment joints. These conditions precluded the use of the conventional copper waterstops and indicated the need of a special material for grouting the shale and for use as waterstops, which would remain plastic and have the ability to come and go with any movement between the abutments and abutting monoliths. Three vertical sealing wells were installed, therefore, along and astraddle of each abutment joint. They were 15 in. in diameter, the two outside wells at each abutment being filled with an internal set-up bituminous material whereas the center well was filled with a mixture of sand and bentonite. At the south abutment the vertical joint between the shale and concrete, as well as the shale itself, was grouted with an internal set bituminous material that was mixed and pumped cold. This material possesses the property of setting up internally, several hours after mixing and injecting, to produce a plastic rubbery product. The shale was grouted with bitumen after the abutting monolith was constructed, the grout being injected by means of a system of piping embedded in the abutting monoliths during construction. Provision has been made for similar grouting at the north abutment should it become necessary.

In discussing the problems of the artesian sandstone upon which most of the high monoliths between the abutments are founded and which underlies the remainder of the dam at varying depths, the author states: "If the dam were built on this sandstone without a grout curtain and without drainage, the base would be subject to uplift pressure decreasing from the heel to the toe * * *" (see heading "Practical Problems of the Artesian Sandstone and the Pink Shaly Sandstone"). This statement is obviously true of any gravity dam regardless of the kind and type of foundation material, unless the foundation materials were absolutely impermeable. The presence in the foundation of an artesian

sandstone which outcrops or has intakes within the reservoir does give rise to an unusual uplift problem. With geological conditions, particularly those of artesian nature which occur at Conchas Dam, it is apparent that: (1) A grout curtain along the heel of the dam is wholly ineffective so far as uplift due to artesian pressure is concerned; and (2) without artificial drainage of the artesian aquifer at the dam, and in the absence of a natural outlet, the artesian uplift will be essentially uniform throughout the entire foundation area and will be equal to the hydrostatic head between pool level and foundation level. To relieve artesian uplift throughout the foundation area sixty-six 8-in. drain holes were drilled into the foundation, as mentioned by the author, at approximately 15-ft centers along the center line of the operating gallery, which is situated approximately 50 ft down stream of the grout curtain at the heel. These holes penetrate the upper artesian sandstone to within 2 to 5 ft of the underlying lower red shale. These drain holes and an inter-connecting tile and gravel drain laid upon the foundation will also serve to dispose of reservoir water passing the grout curtain at the heel of the dam, and will reduce uplift pressure resulting from it.

That artesian aquifers are compressible and elastic to varying degrees, dependent on the kind of rock or material comprising the aquifer, is known and generally accepted by authorities on ground water.[9] However, the nature of this phenomenon is such that any expansion or contraction, due to fluctuations in the artesian head resulting from variations in pool level in the artesian sandstone underlying Conchas Dam, should be fairly uniform throughout the area of the dam. The drainage system previously discussed will also be instrumental in reducing (if it does not wholly prevent) any tendency of the artesian aquifer to expand or contract by maintaining the artesian pressure under the dam at a uniform head.

In addition to the grout curtain which is continuous from the southeast end of the South Wing Dam, through the Main Dam, to the north end of the North Wing Dam, there was an extensive program of consolidation grouting in the canyon sandstone abutments. In this program every joint or crack of any appreciable size, exposed in the foundations or abutments, was cleaned out by washing and tapped with pipes to permit consolidation grouting after sufficient concrete or fill had been placed to permit the use of effective grouting pressures. Consolidation grouting was performed, whenever possible, in advance of the drilling and grouting of the grout curtain, in order that the cores obtained from the holes of the grout curtain might serve as a check on the effectiveness of the grouting. In addition to the consolidation grouting at the south abutment the vertical face of the canyon sandstone receives a protective coating of gunite for a distance of approximately 125 ft up stream from the dam. This is the only area adjacent to the dam where the highly jointed canyon sandstone is not to be covered by back-fill and would be exposed to the waters of the reservoir. In the remainder of the up-stream areas adjacent to the abutments the canyon sandstone, and the upper red shale, are protected from weathering, and the path of any possible seepage is lengthened in a somewhat unusual

[9] "Compressibility and Elasticity of Artesian Aquifers," by O. E. Meinzer, *Economic Geology*, Vol. 23, No. 3, May, 1928, pp. 263–291.

manner by virtue of extensive impervious back-fills. These back-fills extend approximately 160 ft up stream along the abutments from the dam and are themselves protected by pervious and rock-fill blankets.

In addition to past investigations of foundation conditions and allied problems, extensive preparations are being made to investigate: (1) The rate and distribution, as well as the ultimate future, of foundation settlement; (2) the quantitative effects of compressibility and elasticity of the artesian aquifer under operating conditions; (3) the deflection and tilting that will occur, and its relation to settlement; and (4), the effect of a full reservoir on artesian and ground-water conditions. To this end special equipment consisting of tilt-meters, plumb-bobs, underground bench-marks, and triangulation stations are installed in the Main Dam, and observations are made of artesian head at various core holes. The physical characteristics of the lower red shale are investigated by means of virgin samples obtained from several 8-in. drain holes which are extended into it. An extensive program of differential leveling and observing reference points set at the heel and toe of the various monoliths has been conducted continuously from October, 1937, to date (1939) in order to obtain foundation settlement data during construction.

In closing this discussion the writer wishes to emphasize the fact that the difficulties and problems at Conchas Dam due to geological conditions were clearly recognized from the inception of the project and have been met by the continuous application of geological and engineering knowledge. Contributions to geological studies and to the solution of the various foundation problems at Conchas Dam have been made by E. G. Woodruff, who served as Geologist during the period of initial explorations; C. B. Theis, Associate Geologist, U. S. Geological Survey, Ground Water Division; W. M. Rau, Associate Geologist, U. S. Engineer Office, Little Rock, Ark., District; and the author. The writer has served as Foundation Engineer on the project since November, 1936.

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DISCUSSIONS

EARTHQUAKES AND STRUCTURES

Discussion

BY MESSRS. R. S. CHEW, JACOB J. CRESKOFF, AND ARTHUR C. RUGE

R. S. CHEW,[12] M. AM. SOC. C. E. (by letter).[12a]—It is thirty years since the writer offered a paper which dealt with this subject.[13] The analysis is crude and has several apparent errors; and yet the treatment of the stiff structure is similar to that suggested in Section II of the present paper, under the heading "Fundamental General Rule."

The analysis of the flexible structure was not attempted by the writer except to indicate reversal curves for both a uniformly flexible structure and the flexible first story. In later years the writer has realized that the foregoing treatment was insufficient and has made an analysis of the problem assuming the structure to have its base oscillating horizontally, with simple harmonic motion, and has published a monograph on this subject.[14]

From the foregoing it will be appreciated that the writer has read the rigid analysis presented in this paper with keen interest and believes it will be of great value to the profession in its study of the problem. The writer hopes that the treatment will be extended so that it will reveal the condition in which $\frac{T_0}{T}$ varies from 1.6 to 4.0 because he believes in a zone of $\frac{T_0}{T}$ between 1.6 and 4.0. The characteristic curves will appear as reverse curves with nodal points. This is necessary in the consideration of higher buildings in which periods of 1.35 to 1.8 are usual. In the writer's treatment[15] he has attempted to indicate the deformations in this zone.

Turning from theory to a practical consideration, it is to be noted that the value of T_0 is ambiguous.

NOTE.—This paper by Leander M. Hoskins, Esq., and John D. Galloway, M. Am. Soc. C. E., was published in December, 1938, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1939, by Messrs. Homer M. Hadley, and R. McC. Beanfield.

[12] Cons. Engr., San Francisco, Calif.

[12a] Received by the Secretary January 23, 1939.

[13] "Effect of Earthquake Shock on High Buildings," by R. S. Chew, *Transactions*, Am. Soc. C. E., Vol. LXI, December, 1908, p. 238.

[14] "An Approximate Measure of Earthquake Effect on Framed Structures," by R. S. Chew; published privately by the author at San Francisco, Calif., 1938; a copy has been filed for reference in Engineering Societies Library, 33 W. 39th Street, New York, N. Y.

[15] *Loc. cit.*, p. 82.

The authors state that Equation (14) may give some indication of the order of magnitude of period for a tall building. On the other hand, it is impractical to evaluate $E I$ in a building when walls (fire-proofing), partitions, and frame must all be considered as acting.

In the thirty-three years since 1906 the writer has observed the rebuilding of San Francisco, Calif., with structures having a marked difference in the amount and type of bracing, and has made a relative study of measured periods of the tall buildings ranging from seventeen to twenty-nine stories in this city. On the basis of this study he has reached the opinion that at least 50% of the stiffness in a wind-braced, wall-enveloped, tall building, with usual office windows, is due to wall and fire-proofing, etc.

From records of the Japanese it was found that the period of a structure lengthens during a base movement due to the loosening and cracking of the non-elastic materials—and from measurements before and after an earthquake, it has been stated[16] that the increase varies from 30 to 50 per cent. This indicates that the structure passes through several period states until the various resistances produce a period that maintains. Consequently, the period T_0 must be assumed from comparison with existing structures of known period; or it must be based on some empiric formula.

The foregoing indicates why the writer is unable to subscribe to the authors' practical rule of

$$F = m\,\alpha = \frac{W}{g}\,\alpha \qquad (51)$$

when applied to all buildings, stiff or flexible. In fact the writer does not believe that any constant seismic factor is logical.

When applied to the frame the factor produces increased strength but only a small margin of stiffness, whereas stiffness is the required feature. It is understood that strength is necessary; but for the present problem it is sufficient to have a resilient body under the action.

For the purpose of illustration consider the practical problems involved in a six-story design and a twenty-five-story design, assuming the following factors as known and as actually resulting from earthquake acceleration equal to 3.2 ft per sec²: Amplitude = 1 in.; and, period T = 1 sec. Both structures are to be designed under a specified load of $\frac{1}{10} W$ as a lateral force. The designer of the six-story structure will apply the factor giving the structure the necessary strength, but is likely to have a period T_0 of between 0.6 and 0.8 sec, in which case the structure actually has stresses 40% higher than called for by the factor.

Assuming that it is practical to apply the lateral force to the twenty-five-story building, the designer would have to add approximately 40% to his steel tonnage in order to meet the strength requirement; but it will be found that unless diagonal bracing is incorporated, the increased stiffness is not in proportion, and from the writer's study, the period would be decreased 10% to 15% below that of the 20-lb, wind-braced, structure.

[16] "Tests on Maronouchi Building in Tokio," by Tadasi Taniguti, *Paper no. 645, Transactions*, World Power Conference, Vol. 8, 1929.

It will be seen that the structure has changed its flexible characteristics very little, so that its deformation is approximately the same, in which case, although the building is stronger, it will probably suffer as much wall and partition damage as it would under a 20-lb wind design. These results are logical because the factor ($0.1W$) was based on a rigidity, which neither design possessed, under the action described. The stiffer building received the accelerated movement, increased by additional acceleration due to yield. The flexible structure absorbed the accelerated movement in its deformation.

From the writer's viewpoint the requirement by ordinance, for the foregoing, should have been: Acceleration = 3.22 ft per sec²; and amplitude = 0.25 in., 0.50 in., or 1.0 in., depending on the soil, with corresponding periods, T, equal to 0.5 sec, 0.7 sec, or 1.0 sec, in which case the designs would have been more consistent.

Resonance.—In high flexible buildings the danger of resonance is remote due to the fact that an earthquake period is rarely, if ever, as great as the period of the loaded frame with damping removed.

In low stiff buildings $\left(\frac{T_0}{T} < 1\right)$ there is a danger of the period lengthening until it is approximately equal to T.

Damage.—There is no doubt that wall damage is unavoidable due to deformation except in such low structures (as to height and design) that their maintained period is less than 0.5 sec, in which case deformation is small.

JACOB J. CRESKOFF,[17] ESQ. (by letter).[17a]—Under the headings "Dynamical Theory as a Guide to Practical Design: Fundamental Practical Rule" and "Danger of Resonance," the authors express some interesting conclusions. Since m equals $\frac{W}{g}$, the fundamental practical rule of the authors may be restated as:

$$F = W\frac{a}{g} = W\,K \qquad (52)$$

in which, W = weight, in pounds; and, K = a constant, varying from 0.02 to 0.10. Hence, the fundamental practical rule of the authors will be recognized as the basis of the statical method of aseismic design. Naturally, statical design is to be preferred to a system that disregards aseismic features. However, it should be kept in mind that, although some statically designed structures have reacted well during earthquakes, it has become increasingly apparent that survival in these cases has been largely a matter of chance.

The following factors are involved in most of the earthquakes that occur in the United States: Invisible elastic earth waves are generated by the slipping of contiguous fault-blocks. Upon reaching a certain location, these waves jar the site and its superimposed structures into vibration. The response of a particular building is determined by the complex relationship between the earth waves, the site, and building characteristics, but principally by the resonance or synchronism ratio between the dominant periods of the site and building.

[17] Cons. Engr., New York, N. Y.

[17a] Received by the Secretary February 14, 1939.

If the ratio approaches unity, the building will vibrate dangerously within a few seconds—only several vibrations. That synchronism is of profound importance in aseismic design has been demonstrated not only theoretically, as in the paper by the authors, but experimentally by Arthur C. Ruge, Assoc. M. Am. Soc. C. E.,[18] and others, and by analyses of actual earthquake damage. A. Blake has shown[19] that the periods of invisible earth waves may be of any values included in the entire range of building periods. One or more dominant periods may appear in a site vibrating by reason of the passage of these waves through it. Finally, the superimposed structure vibrates in a number of natural periods.

At first glance therefore it seems improbable that a practical rule for estimating the probable effects of synchronism can be formulated. However, the writer is of the opinion that (except for the improbable case of synchronism between the earth waves and the site) the degree of synchronism existing between the dominant site and building periods is a reliable index to the probable extent of stresses due to synchronism.

The dominant periods of a site can be determined by jarring it with dynamite blasts, or with a shaking machine, recording the vibrations by seismographs, and interpreting the records. The dominant periods of an existing building can be found by recording and interpreting its vibrations as caused by wind, or with a shaking machine. In the case of a projected building, the writer agrees with the authors that a designer is powerless to estimate the possible danger from synchronism unless he can compute the periods of vibration of the building. Fortunately, there is now (1936) available a wealth of observational data on building vibrations.[20] With the plans and loading of the building, the empirical data, and the available vibration equations, the designer can compute the periods of the projected building with reasonable accuracy.

For example, suppose that, as in the statical method, the horizontal force to be resisted by a building is assumed to be acting at right angles to the elevations of the building, at the foundation, floor, and roof levels, and calculated by the formula

$$F = W K \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (53)$$

in which F is the horizontal force acting at the section under consideration, in pounds; K is a numerical constant depending on dynamical and experimental considerations (see Table 2); W is the total dead load plus actual live load at,

TABLE 2.—VALUES OF K

Site Material	Values for the Following Synchronism Ratios, $\frac{T_b}{T_g}$:									
	2.0	1.8	1.6	1.4	1.2	1.0	0.9	0.8	0.7	0.6
Bed-rock	0.05	0.06	0.07	0.12	0.21	0.30	0.25	0.12	0.07	0.05
Consolidated ground	0.05	0.07	0.08	0.13	0.23	0.33	0.28	0.13	0.08	0.05
Soft ground	0.05	0.05	0.06	0.09	0.15	0.22	0.19	0.09	0.06	0.05

[18] "Earthquake Resistance of Elevated Water-Tanks," by Arthur C. Ruge, *Proceedings*, Am. Soc. C. E., May, 1937, p. 801.

[19] "The Recording of Strong Seismic Motion," by A. Blake, *Bulletin* of the Seismological Society of America, July, 1933, p. 111.

[20] *Bulletin No. 201*, U. S. Coast and Geodetic Survey.

and above, the section under consideration, in pounds (minimum live load to be one-fourth the design live load); T_b is the period of vibration of the building, in seconds; and T_g is the period of vibration of the site, in seconds. The synchronism ratio $\frac{T_b}{T_g}$ that gives the largest value of K in the plane of vibration under consideration is to be used. The values of K given in Table 2 must be divided, however, by the order of the period of vibration (that is, the fundamental period will be divided by (1), the first harmonic by (2), etc.). The minimum value of K used for any building will be $K = 0.05$, except for public buildings, for which K is not less than 0.10.

The seismic force, $F = W\,K$, acts against the mass of a building. Assuming that the mass is concentrated at the floor levels, and that floors are horizontal girders that deflect as units, then the columns and beams between two consecutive floors may be regarded as vertical beams—deflecting equally because of a concentrated horizontal force acting against the upper floor—and with end fixities determined by the degree of fixity between the vertical beams and the floors. Under these assumptions, the seismic force is distributed to all of the vertical beams, and to each in proportion to its relative rigidity.

It will be noted that the values of K given in Table 2 are frequently much larger than the maximum value described by the authors under their fundamental practical rule. This is to be expected, of course, since the values of K in the table reflect the influence of synchronism between site and building periods during an earthquake.

One might judge that the use of such high values of K would add materially to the cost of a structure. This might be the case if the frame of the building is to be designed for these large values of K. However, if, in addition to the frame, the walls and floor and roof slabs are designed to resist the seismic force, it has been the experience of the writer that these higher values of K will not affect, appreciably, the cost of the building as a whole.

ARTHUR C. RUGE,[21] ASSOC. M. AM. SOC. C. E. (by letter).[21a]—This interesting paper draws attention to the great difficulties in the way of rational design against earthquakes. Certainly it is clear that a purely analytical approach to the problem is out of the question, except for such general and valuable qualitative information as that developed by the authors and several other writers on the subject. The natural tendency of the average designing engineer is to throw up his hands at the thought of making any dynamical analysis at all, and it is only through the persistence of these investigators that the subject is kept alive and with it the hope of an ultimate solution.

The writer would like to comment upon a few statements contained in the paper with a view to further clarification, rather than criticism, and to add some comments on the interpretation of forced vibration analyses like those presented by the authors.

It is clear that the authors recognize the importance of the transient response of structures, and that they regard the forced or steady-state response as only

[21] Asst. Prof. of Eng. Seismology, Mass. Inst. Tech., Cambridge, Mass.

[21a] Received by the Secretary February 16, 1939.

a guide to judgment in design. There is one important point, however, which seems to be quite generally overlooked in connection with the behavior of the higher modes of vibration. It will be found invariably that a considerably closer "tuning in" of earthquake frequency with natural frequency is required in the higher modes for the same degree of dynamic magnification; the higher the mode, the sharper the tuning must be.

This fact is well known to those who have had experience in generating the higher modes of vibration of mechanical systems or models. Many casual experimenters have had the mistaken notion that the effect is due to a greater degree of damping in the higher modes. Although in some cases there may be greater damping and an attendant reduction of magnification in the higher modes, the presence of damping will always tend to make the tuning less critical at resonance which, therefore, makes it impossible to explain the tuning phenomena observed on the basis of damping effects.

Physically, this means that the probabilities of damage in the lower modes are far greater than in the higher, simply because the degree of tuning is less critical in the lower modes. As an example, consider the ratio $\frac{V}{V_0'}$ (Equation (18*c*)) at the base of the beam for the first and second modes; that is, the ratio of the actual shear at the base to the shear that would occur in a perfectly rigid structure of the same weight. The following data were obtained from Equation (18*c*):

Mode	$\frac{T_0}{T}$	$\frac{V}{V_0'}$
First	0.95	7.0
Second	0.98	4.6

That is, the earthquake period is 5% above the first mode resonance and 2% above the second mode. It is apparent that the tuning in at the second mode must be more than $2\frac{1}{2}$ times as sharp as at the first or lowest mode in order to produce the same dynamic magnification. The effect becomes even more pronounced above the second mode. Now, it must be remembered that the foregoing data are not the actual shears but magnification factors—the shears are $V = 7.0\ V_0'$ and $4.6\ V_0'$; and it is here that a second and equally important consideration enters; namely, the actual values of V_0' decrease very definitely at the higher earthquake frequencies and hence at the higher modes of buildings. Exactly what law governs the variation of earthquake amplitude with frequency is not quantitatively known as yet, but the fact that the amplitudes do decrease with increase in frequency is too well known to be debated. Over very limited ranges the trend may be reversed; but when widely separated ranges are compared (as the first and second modes of a structure which may bear a 5 : 1 or a 6 : 1 period relation) one may confidently expect to find a large drop in amplitude in the range of the higher modes.

The natural conclusion, therefore, seems to be that the modes above the first (or, in rare structures, possibly the second) are not nearly so important as would be indicated by a superficial study of the steady state analysis. It is on the basis of such reasoning that the writer questions the statement (see

Part II, under "Dynamical Theory as a Guide to Practical Design: Danger of Resonance") that: "Since any given structure has a multiplicity of natural periods, and since the harmonic analysis of seismograph records has shown a considerable range of values of T, it seems impracticable to formulate any simple rule for minimizing the probability of dangerous stresses due to resonance." It seems to the writer that by limiting the modes to be studied to one or, perhaps, two, the problem is not so hopeless as it might at first seem. The mere fact that difficulties still beset the investigator should not lessen the determination to achieve a full solution ultimately.

It is further true that the frequency selectivity of the higher modes may be augmented by proper design. Indeed, the introduction of a flexible first story has just such an effect, as may easily be visualized.[22] The only objectionable feature of this construction is that it increases, considerably, the liability of the first story to damage, due to the lowered period of the fundamental. The writer has yet to see a suitable and economically feasible design proposed for the first story of such structures.

The statement (see Part I under "Earthquakes and Their Effects: Relation to Structures"), to the effect that the forces of the moving ground are unlimited, may require some modification. Although it is doubtless true for structures on rock and for light structures on compact soil, there is reason to believe that heavy structures resting on soil do not behave as if the ground motion were inexorable, but instead they influence the ground movement locally to a marked extent. Some evidence of this influence has been noted in the effects of the Japanese earthquake of 1923, and analytical studies by K. Sezawa and K. Kanai[23] lead to the same conclusion. Professor Jacobsen has also presented a valuable study of this phenomenon.[24]

In Part I (under "Earthquakes and Their Effects"), the authors express the limits of earthquake periods as 0.1 sec and 2 sec. This is a much disputed point, however, and there is reliable evidence that wave periods much longer than 2 sec occur in destructive earthquake motion.[25] In general, it appears that the more severe and extensive the quake, the more prominent become the long-period components of motion. This fact is of great importance and should receive careful consideration in connection with ground-period studies made by shaking machines and by observation of small earthquakes and blasts. A study of the Long Beach motion (integrated by the U. S. Coast and Geodetic Survey) shows a surprising amount of long-period motion. Of course, waves shorter than 0.1 sec occur, but the authors probably considered them of no structural importance. Perhaps this question might best be left open at present.

In Part I, the authors omit consideration of the possibilities of the experimental or semi-experimental approach to the problem of anti-seismic design.

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[22] See "Experimentally Determined Dynamic Shears * * *," by L. F. Jacobsen, *Bulletin*, Seismological Society of America, Vol. 28, No. 4, October, 1938.

[23] *Bulletin*, The Earthquake Research Institute, Tokyo Imperial University, Vol. XIV, Part 2, June, 1936.

[24] "Natural Periods of Uniform Cantilever Beams," by Lydik S. Jacobsen, *Proceedings*, Am. Soc. C. E., Vol. 64, No. 3, March, 1938, p. 432.

[25] "Earthquake Investigations in California," *Special Publication No. 201*, U. S. Coast and Geodetic Survey; see Chapter 9 by B. Gutenberg.

The writer believes that sufficient facilities exist for extending the knowledge of quake-resistant design to the point of practical significance by combining the experimental and analytical techniques effectively. The principal reason for the lag of accomplishment behind facilities has been, and is, the lack of adequate funds for establishing the requisite long-time basic research with adequate personnel. To say that the surface has only been scratched is to risk exaggeration, but the profession owes a debt of gratitude to those men who have pushed forward the theoretical analysis until the possibility of a practical solution may be seen.

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DISCUSSIONS

SIMPLIFIED WIND-STRESS ANALYSIS OF TALL BUILDINGS

Discussion

BY SAMUEL T. CARPENTER, JUN. AM. SOC. C. E.

SAMUEL T. CARPENTER,[12] JUN. AM. SOC. C. E. (by letter).[12a]—The statement in the "Synopsis" that the author has presented a simple method of analysis is true, the writer having applied the method satisfactorily to several cases with a great deal of ease, but not without first studying, thoroughly, the principles[3] upon which it was based.

Equation (5) expresses the horizontal resistance of a column for a total lateral displacement of $\Delta = 1$. Not only is this equation important in the analysis, but it may be of equal importance in making the structural design, if the designer places any importance in the final elastic action of his structure and the achievement of initially assigned column and girder shears. Although it is impracticable always to provide the required value of k for the columns of a bent, it is usually practicable to provide approximately the correct stiffness for the girders framing to the columns. For example if, by the cantilever method, the designer has assigned a value for shear equal to P to Column A of the bent shown in Fig. 3, and has provided a column of a stiffness k, then $(f' - f)$ should equal $\frac{h^2 P}{6 E k}$. This criterion, plus Equations (6a) and (6b), when used in conjunction with a desired story deflection equal to Δ, should yield approximately correct values of stiffness for the adjacent girders.

For a case of building design Δ may be contrellod by the designer to be $0.002\,h$, or any other desired value. In most cases of regular story framing, and for an exterior column, k_A could be assumed equal to k_B, making $(f' - f)$—from a simplification of Equations (6a) and (6b)—equal to $\frac{2\,k_A\,\Delta}{k_A + k}$. Then

$$P = \frac{6\,E\,k\,\Delta}{h^2}\left(\frac{2\,k_A}{k_A + k}\right) \qquad (8)$$

NOTE.—This paper by Otto Gottschalk, Esq., was published in December, 1938, *Proceedings*. This discussion is published in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[12] Asst. Prof., Civ. Eng., Swarthmore Coll., Swarthmore, Pa.

[12a] Received by the Secretary February 14, 1939.

[3] *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 1019.

from which k_A could be found and would be approximately the desired stiffness of the girders connecting to the exterior column. A similar treatment could be used in designing the girders framing to the interior columns. It is apparent, of course, that the girder selections must also depend on maintaining a safe combined unit stress.

The designing of a tall building to achieve a certain elastic action was pointed out and described by H. V. Spurr,[13] M. Am. Soc. C. E. Later, in a paper[14] describing the analysis of a model, designed to check experimentally the action of a structure elastically designed, it was shown that such a design satisfied not only the required deflection but yielded experimental shears equal to previously assigned values. This latter paper also deals with the practical problems of correct designing in the lower stories and around points of structural irregularity. The writer believes that the author's method would be thoroughly applicable in design, but would be somewhat dependent upon the choice of correct lengths of columns and girders for the calculation of stiffness.

The author shows that the secondary stresses, from the shortening and lengthening of columns by the wind stresses beyond the points allowing planarity of the floor joints, are of importance. It has been shown elsewhere[15] that in a tall building, the adjustment of column stresses is not serious in the upper stories, but of considerable importance in the lower stories, due to the accumulation of girder shear corrections. The method presented by the author would appear to be applicable in the adjustment of column stresses and the calculation of the secondary girder shears. However, it should be the duty of the designer, particularly in the taller building, to assign correct values of direct stress and shear in his original design, where possible, and then to design the structure so that they are obtained, thus eliminating this adjustment.

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[13] "Wind Bracing," by H. V. Spurr, McGraw-Hill Book Co.

[14] "Tests and Design of Steel Wind Bents for Tall Buildings," by George E. Large, Assoc. M. Am. Soc. C. E., Samuel T. Carpenter, Jun. Am. Soc. C. E., and Clyde T. Morris, M. Am. Soc. C. E., *Bulletin No. 93*, The Engineering Experiment Station, Ohio State University, 1936.

[15] "Wind Stress Investigation Method Considering Direct Deformation of Columns," a Report by George E. Large, Ohio State University Experiment Station, 1937.

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DISCUSSIONS

THE RISK OF THE UNEXPECTED IN SUB-SURFACE CONSTRUCTION CONTRACTS

Discussion

By Messrs. Alonzo J. Hammond, Frederick W. Newton, David A. Molitor, F. B. Marsh, and Evan S. Martin

Alonzo J. Hammond,[3] Past-President, Am. Soc. C. E. (by letter).[3a]—In 1934 a Committee of the Construction Division of the Society, on Substructure Engineering, presented a report which, subsequently, was published as Manual of Engineering Practice No. 8. The object of this work was[4]:

"* * * to establish fair and equitable practice, and to correct undesirable procedures and abuses in the relations between owners, engineers, and contractors concerned with the design and construction of bridge foundations."

It is gratifying to note that the author credits the manual of practice with eliminating "much of the costly uncertainty involved in underground construction, with corresponding benefit to owner and contractor" (see "Synopsis").

He emphasizes the need of carefully worded specifications, but this does not carry with it the placing of all the burden on the contractor, as has been clearly stated. It is now considered best practice to give the engineer means and authority to obtain reasonably full and complete information of subsoil conditions, and to be responsible for such data.

In 1912, while conducting studies preliminary to the design of the Lake Street double-deck bascule bridge for the City of Chicago, Ill., the writer found information of borings made on each side of the river, about 250 ft apart, showing rock at Elevation −75.0 ft. That rock should be at such an exact uniformity of elevation raised a doubt in his mind as to whether it was bed-rock. New borings were ordered and confirmed the occurrence of rock at

Note.—This paper by Oren Clive Herwitz, Esq., was published in January, 1939, *Proceedings*. This discussion is published in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[3] Cons. Engr., Chicago, Ill.

[3a] Received by the Secretary February 6, 1939.

[4] "Engineering and Contracting Procedure for Foundations," *Manual No. 8*, Am. Soc. C. E., 1934, p. 3.

Elevation −75.0, but only 4 in. thick; so the driller continued down and found the bed-rock at Elevation −110.0.

The author has cited a number of border-line cases and the effect, on the Court, of the evidence presented by the engineer, as illustrated in the Fort Smith case (25).[4a] It seems reasonable to reach the conclusion that the engineer must set himself up primarily as an impartial judge, fair to both owner and contractor; otherwise a contestant does not reach conclusive knowledge of the law in the case until he reaches the Court of last resort, and it may not be on equity then.

To cite, briefly, a case in the Chicago district: The contractor was given certain information, but there were some additional borings referring to a material as "very hard," which borings were not shown. About 12 ft below the surface, the contractor found conglomerate rock, formed of boulders and clay cemented together, which had to be drilled and blasted and broken up before it could be shoveled. A favorable decision by the Circuit Court (74) in this case was later reversed by the Court of Appeals (75).

Engineers, generally, would consider the decision of the Circuit Judge in this case as representative of good law and common sense as against the decision of the Appellate Court.

A contrary case of a decision of a lower Court being confirmed by a higher Court on a rule of reason is that of Schiltz *vs.* Akers, *et al.*[5] Action was brought by the plaintiff to recover the sum of $7 550, alleged to be due from defendants under the terms of a written contract for drilling of oil wells. The contract provided, among other things, "that in the event [and this is the important feature of the contract] rock is encountered in drilling any of said wells 'plaintiff agrees to drill one foot through the same without extra compensation, but beyond that defendants agree to pay $75.00 per day extra compensation for each day of nine hours required in drilling through all rock formations.'" The lower Court ruled in favor of the plaintiff, and the decision was confirmed by the Supreme Court (76).

As a young man the writer had a feeling of awe for the Supreme Court, until a very able lawyer tempered it by the remark that, although law in itself was an exact science, the interpretation by the Supreme Court was by some very human individuals. Contrary to the legal technicalities involved in some of the cases mentioned in the paper, the writer can cite the following as a common sense solution of a very difficult problem: On the main drainage canal from Chicago to Lockport, Ill., one firm of contractors had Sections 2, 3, and 4 at Lemont, Ill., the original contract calling for 27 cents and 28 cents per cu yd for glacial drift. The indurated clay was found to be almost as difficult to excavate as rock; and, therefore, by mutual consent, the contract was re-advertised and the bids of four contractors ranged from 59 cents to 75 cents per cu yd for the same sections. The Chicago Sanitary District, however, finally made a new contract with the original contractor on the basis of 50 cents and 49 cents for Sections 2 and 4, respectively. There were, perhaps, two

[4a] For references to numerals in parenthesis see "Court Citations and References" at the end of the paper and this discussion.

[5] Supreme Court of California, Vol. 292, Pacific Reporter 463, October 20, 1930.

valid reasons for a renewal of the contract on the terms mentioned. The Trustees, no doubt, were willing for the contractors to recoup some of their losses, whereas the contractors, having their plant on the ground, were willing to make what appears as a low price compared to the bids of the other contractors.

The author has presented a most valuable list of case material for study by every engineer and contractor engaged in sub-surface work.

Court Citations and References.—

(74) The Circuit Judge said, in his decision favorable to the contractor: "I am led to believe that these prior investigations could have been obtained by the defendants and, in every sense of the word, should rightly have been in the possession of the complainants so they might be correctly guided in their proposal and estimates on the material to be handled. The suppression of all such prior information unquestionably worked a hardship, if not a fraud, on the complainant." (Citation Vol. 89, Federal Reporter 251.)

(75) The Court of Appeals reversed the lower Court, stating: "Trustees are not bound to exercise diligence to obtain information concerning the nature or cost of work for the benefit of bidders with whom they deal at arms length * * * and a contractor, for the excavation of a section of the drainage canal, is not entitled to a rescission of his contract because he encountered a substance more difficult and expensive to excavate than anything he was led to expect from an examination of profile and data in the office of the Chief Engineer."

Furthermore, there was "No provision in statute * * * which required that information concerning the nature of the materials to be excavated should be collected for the benefit of the bidders, and that such duty on the general principles of law or equity there is no foundation in authority or reason." (Citation 91, Federal Reporter No. 511, February 7, 1899.)

(76) The Court declared as follows: "On this appeal, defendants strenuously contend that the evidence shows that plaintiff drilled the greater portion of the time through shale and that the terms 'rock' and 'rock formations' do not include shale. To support this contention, defendants produced a petroleum engineer who testified that technically rock includes 'shale' or 'stone,' but in the oil field parlance the term refers to hard formations, usually in large bodies as, for instance, 'granite,' and further that 'shale' is not considered rock by oil people. The trial court interpreted the term used in the contract in its general sense and refused to interpret it in the limited fashion suggested by the expert. We agree with the interpretation of the trial court * * * the term 'shale' is defined as rock formed by the consolidation of clay, mud or silt, having a finely stratified or laminated structure * * *. Under the circumstances, the general and popular meaning of the term is reasonably intended."

Frederick W. Newton,[6] Assoc. M. Am. Soc. C. E. (by letter).[6a]—An ideal toward which the engineer should strive in the preparation of a construction contract. The engineer is required to work with the tools at his disposal, however, and too frequently these do not include the complete investigation of sub-surface conditions which he would like to have. The

[6] Attorney and Counsellor at Law (Wait, Wilson & Newton), New York, N. Y.

[6a] Received by the Secretary February 14, 1939.

complete investigation that should be made by an engineer before advertising a contract means more than simply making such borings and soundings as are necessary to determine the stability of the structure. It is recognized that in some cases such complete investigation is impossible.

The value in Mr. Herwitz's paper is that it helps to show how, in preparing a contract, the engineer may make the best use of such information as is available to him. The paper places too much emphasis on the wording of specifications, however (particularly the so-called "exonerating" clauses), and not enough on the facts. The statement in the "Synopsis" that "Experience in this type of litigation shows that emphasis is placed on carefully worded specifications, often with results expensively significant to one party or the other" is, to say the least, subject to question. Mr. Herwitz presents many instances where, in two or more cases involving contracts with the same or similar specifications, the result has been that in one the contractor was awarded increased compensation and in others the increase was denied.

The fact is that the Courts have endeavored to decide each case on its merits. In practice, a case is first decided and then an opinion is written explaining the decision. In most of the cases the exonerating clauses have been urged as a defense. Hence, these clauses are discussed in the opinions of the Court but, with the exception of a few cases in which there have been provisions of the contract or specifications relating to the particular matter in suit, it will be found that the facts, not the contract clauses, form the real basis of the decision.

The author recognizes this fact in his "Conclusion." He states that, in the last analysis, each case must rest upon its own distinctive facts. That being so, a discussion of this subject should direct attention to the facts that have led to litigation in the past, in order that a repetition of those facts may be avoided in the future. No exonerating clause can be drawn which will avoid the result of carelessness, negligence, or deception in the preparation of a contract.

Contract Should Describe Work Correctly.—The first and most important rule for the engineer who is preparing a contract is that the work to be done by the contractor must be described, clearly and accurately, in the contract. A bidder looks first at the "description of the work" and next at the bidding sheet. He makes a proposal for doing the work which the contract describes. If he is required to do more expensive work, he is usually entitled to increased compensation.

Where the contract described the work as "earth excavation," the contractor was entitled to additional compensation for excavating rock, Maney *vs.* Oklahoma City (2)[6b]; a contract that called for the excavation of earth, hard-pan, rock, etc., and defined each of these materials, did not include the excavation of adobe or gumbo where it was shown that these materials were not included in the definitions, Sweeney *vs.* Jackson County (3); a contractor who had undertaken to construct a tunnel in rock was entitled to additional compensation for a part of the tunnel that was in loose earth and water and required

[6b] For reference to numerals in parenthesis, see "Court Citations and References," in the appendix of the paper and at the end of this discussion.

the use of compressed air, McGovern *vs.* City of New York (19); a contractor who had agreed to construct a tunnel in free air received increased compensation because he had to construct it in compressed air, Montrose Contracting Company, Inc. *vs.* County of Westchester (66); and, a contractor who had undertaken to construct a tunnel in solid rock with gunite lining received additional compensation because he was required to excavate the tunnel in loose rock and to place a monolithic concrete lining, City of Dallas *vs.* Shortall (34).

The cases in which the contractor has received additional compensation are not the only ones that show the importance of a clear and accurate description of the contract work. Other cases in which the owner has ultimately succeeded are equally persuasive because, if the engineer had been sufficiently careful in the preparation of the contract, the owner would have been saved the annoyance and expense of long litigation. In the case of Dean *vs.* Mayor of New York (4) the contract described the work as the grading of Dyckman Street from the Hudson River to Exterior Street. The drawings showed only a part of that work. It required litigation that went to the Court of Appeals to determine just what work the contractor was required to do. Dickinson *vs.* City of Poughkeepsie (3) is one of many cases that involve the question of whether a contractor for earth excavation is required to excavate hardpan. This difficulty may be overcome by taking bids for all excavation without classification, or by a sufficiently broad definition of "earth excavation," or by requiring that the contractor bid a separate unit price for hardpan excavation. In Barash *vs.* Board of Education (4) there would have been no lawsuit if the architect had been more careful to specify, fairly, the work that the contractor was required to do. The contract required the installation of Kalameined windows in all openings from a light shaft in an existing building. It was not clear whether this referred only to openings that the contractor was to make or to all openings, including those which already had windows.

Bidder Must Inspect Site.—The foregoing does not mean that a bidder may accept the description contained in the contract and "close his eyes" to facts that he knows or should know and which would demonstrate that the description is inaccurate. In Lentilhon *vs.* City of New York (4) the contractor undertook to remove the embankment forming the old reservoir at 42nd Street and Fifth Avenue, the site later occupied by the New York Public Library. He figured the contract on the basis of the slopes of the embankment shown on a plan which was not made from survey and not drawn to scale. He did not attempt to ascertain what the actual slope was although he might easily have done so. In Jahn Contracting Company *vs.* City of Seattle (34) a contractor for a sea-wall blindly relied upon a drawing made long before the grading of adjacent streets. A mere walking over the job would have shown that at the time of bidding the streets had been graded and that the line on the plan was wrong.

Representations Concerning Conditions Affecting Work.—After the contract provisions describing the work to be done by the contractor, the next subject is the information given to bidders concerning conditions that may affect the cost of the work. Such information may relate to subsoil conditions determined

by soundings or borings, or to the sources of supply of materials, or to other conditions affecting the particular job.

It is recognized that, in general, the giving of information to bidders results in lower bids for the contract work because if bidders do not have the information, they are certain to make liberal allowances for contingencies. It is recognized, also, that engineers are subject to the temptation to make the contract appear attractive to bidders. In facing this situation the Courts have adopted certain rules which are undoubtedly fair and equitable. The difficulty arises in the application of these rules to the great variety of facts presented.

Borings.—It will probably be most helpful to discuss first the matter of borings. If borings have been made, bidders for the work should have the advantage of such information as those borings disclose. The question for the engineer is how this information should be given to bidders.

When engineers use borings, they draw lines from one boring to another and assume that between the borings the material will be substantially the same as that indicated by the borings. The Courts have recognized that this assumption is not always in accord with the fact. The Court ruled in Elkan *vs.* Sebastian Bridge District (25) that the giving of borings is not a warranty of the condition of the subsoil between the borings; and that all the owner warrants is that the borings have been correctly shown. In Groton Bridge and Manufacturing Company *vs.* Alabama and Vicksburg Railway Company (24) the contractor for the construction of bridge piers complained that the material encountered was unlike the material shown by borings which had been made near but outside the location of the piers. The borings had been plotted correctly, but there was a dip in the strata between the borings. The contractor did not recover.

In Detroit & I. Ry. Co. *vs.* A. Guthrie and Company, Inc. (28) the specifications stated that the borings showed blue clay in a borrow-pit 15 ft below the surface. The contractor found it much closer than that to the surface, but he failed to show that the borings were in error or that at the site of the borings the clay was less than 15 ft from the surface. The Court ruled that the fact that clay was found near the surface was some evidence that the borings were erroneous, but that, standing alone, such evidence was not sufficient.

The engineer may safely give to bidders all of the boring information in his possession, provided it is given accurately and without concealment. Among the cases cited by Mr. Herwitz in which this had been done and in which the contractors were not allowed to recover increased compensation are Foundation Company *vs.* State of New York (7); Pearson & Son Inc. *vs.* State of New York (8); and The Arthur A. Johnson Corporation *vs.* City of New York (10). A similar situation is presented in Murray *vs.* State of New York (8) but with the added fact that by reason of prior contracts in the same vicinity the contractor was entirely familiar with conditions and was not in any way misled by the borings. Irving Trust Company, as Trustee, *vs.* City of New York (21) presented an instance in which the boring information had been given to the bidder exactly as it was received from the boring contractor. The borings were inaccurate but the Court found that there was no proof

that the city was aware of any condition different from what the borings showed, and disallowed the contractor's claim.

A clear illustration of the advantage of telling bidders just what preliminary investigation has been made is found in Kelly *vs.* City of New York (33) in which the contractor was denied a recovery notwithstanding the fact that the "probable surface of rock" shown on the drawings was incorrect. The contract described, correctly, the preliminary investigation that had been made, stating that some rod soundings had been made and the results were shown on the plan but were not warranted to be correct. This told the contractor exactly what had been done. The earth cover was not deep and he could have checked the soundings if he had cared to do so.

An engineer who furnishes borings or boring information to a bidder must exercise care that he furnish all of the information without distortion or concealment. If there has been distortion or concealment by which the contractor has been damaged, he is entitled to recover his damages. Cases of this kind are Christie *vs.* United States (13); United States *vs.* Atlantic Dredging Company (51); Jackson *vs.* State of New York (15); Stewart and Company, Inc. *vs.* State of New York (16). In the Christie case (13) the engineer who recorded the results of the borings had neglected to record logs and stumps encountered, which he considered of no importance. In the Jackson case (15) the State showed the results of one set of borings, but failed to show a set made some years earlier which indicated hard material. In the Atlantic Dredging Company case (51) the borings were given correctly but they did not penetrate to the full depth of the excavation required; and, although the engineer knew that the reason for not carrying the borings deeper was that they had encountered hard material, he nevertheless put in the contract the statement that the material was believed to be mainly mud, or mud with an admixture of fine sand. In the Stewart and Company case (16) the State gave information indicating soft material but withheld other information indicating hard material.

In these cases the giving of erroneous information or the withholding of information was not necessarily done intentionally or with fraudulent motive. In fact, in the Christie case (13) there was no question that the engineer honestly believed that the information concerning the logs and stumps was of no importance.

Statements Concerning Source of Supply of Materials.—The cases relating to the source of supply of materials for highway work show again that the decisions are based on the facts of each case rather than on exonerating clauses of the contract.

In Gross and Son *vs.* State of New York (49) the representation was made on the contract drawings that stone would be found in a ledge near a certain station. By a provision of the contract the contractor warranted that he had personally investigated the local sources of supply. He had made no such investigation and when he went to look for the stone after the contract was made he found a level field with no sign of stone. He wrote to the engineer that he had "assumed" that the information on the plan was correct. He

had no right to make such assumption in the face of the provision which required that he personally investigate that particular matter.

In Atlanta Construction Company *vs.* State of New York (50) the contract contained a similar statement and the contractor recovered additional compensation because afterward it developed that the land on which the stone was located was in the possession of a life tenant who did not have the right to sell the stone. The difference between this case and the Gross case (49) is that in this case the bidder had done what the contract required; he had investigated and had found the stone. The Court found that there was a mutual mistake but one for which the State was responsible by reason of its direction and statement in the contract.

Semper *vs.* Duffy (23) and Brennan Construction Company *vs.* State of New York (49) are cases in which the State merely gave to bidders the reports of the probable location of materials, made by its field engineers to their superiors. The Court said that such reports were "suggestive merely, directing him (the contractor) for inquiries to what the State considered the nearest available source of supply."

An instance of a special provision that proved a bar to a recovery by a sub-contractor occurred in Kuhs *vs.* Flower City Tissue Mills Company (48). The contract provided that good gravel would be found in the excavation, but if sufficient gravel were not found, the sub-contractor should furnish gravel of equal quality. There was some gravel but not enough. In obtaining gravel from other sources the sub-contractor had done no more than he had agreed to do.

Other Cases Involving Representations.—Other cases in the group characterized by lawyers as "breach of warranty" cases show the necessity for both care and good faith in the making of statements or representations which may affect the cost of the work.

The leading case is Hollerbach *vs.* United States (45). The facts are stated in Mr. Herwitz's paper. The lesson for engineers is that no general exonerating clause will overcome the effect of a positive statement negligently or carelessly made in a contract. Another application of the rule is found in Delafield *vs.* Village of Westfield (46).

In the two foregoing cases the engineers did not know that their statements were wrong; but the statements had been made carelessly. Passaic Valley Sewerage Commissioners *vs.* Holbrook, Cabot and Rollins Corporation (17), the facts of which are stated by Mr. Herwitz, presents a situation in which the engineers did know that the borings did not disclose the true condition. They knew that the presence of the cemented formation was essential to the prosecution of the work; and, they knew that in the two prior attempts to build the tunnel this formation had not been found. Their attempt to shift the burden to the contractor received from the Courts the treatment which it deserved.

Representations as to Ground Surface Elevations.—There are two cases in which the facts are very similar; in one the contractor recovered; in the other the architect and Board of Education felt that the contractor should recover, but they were overruled by the Courts. The first of these cases is Pitt

Construction Company *vs.* City of Alliance, Ohio (37) in which the Court held: That the contractor was entitled to use the ground-surface elevations given on a detailed plan for the work; that he was not required to make a survey to determine that those elevations were correct; and, when it was found that they were wrong, that he should have additional compensation. In the other case, T. J. W. Corporation *vs.* Board of Higher Education of the City of New York (52) there was a similar error in the elevation of ground surface given on the plans; but here the Court ruled that the contractor could not recover. The reason for the difference in the two decisions is that in the Pitt Construction Company case the contract contained no reference about checking the elevations although it did contain the usual clause in regard to the contractor's inspection of the site. In the T. J. W. Corporation case there was an express provision in the contract which required the contractor to check the grades.

A similar result was reached in Conners *vs.* United States (35) where the contract showed approximate rock levels but contained a provision that the bidder must satisfy himself as to the accuracy of these levels and as to rock surfaces in general.

Equality of Opportunity.—Throughout many of the cases on this subject there runs the question of "equality of opportunity." If conditions are such that the bidder's opportunity to know them is as good as that of the owner, the Courts hold that the bidder should have taken advantage of that opportunity.

In Odell *vs.* City of New York (6) the contractor complained of an excess quantity of water encountered in the construction of a part of the New York City Aqueduct. The Court found that not only was there no representation of the quantity of water that would be encountered, but also that at the time of bidding, test pits along the line were open and the contractor had as much opportunity as the City to estimate the quantity of water. It did not appear that either party had made any attempt to measure the flow. In Faber *vs.* City of New York (22), on the other hand, the Court held the City responsible for an error in the elevation of rock, as shown by its borings, on the ground that the rock surface was far below the bottom of the river and that the contractor could not have been expected to make borings.

It should be noted, however, that a provision that the contractor must inspect the site does not require that he make borings or other investigations to determine that positive information given him in the contract is correct. For example, in the Hollerbach case (45) the court remarked that the contractor might have ascertained the true nature of the material back of the dam by the simple device of making rod soundings, but in view of the positive statement in the contract, he was not required to do so.

The rule seems to be that a contractor may rely upon the accuracy of the grades, including rock-surface elevations, given him in the contract; but if the contract expressly requires that he check such grades or elevations, and if the conditions are such that he might reasonably have done so before bidding but he has failed to do so, he should not complain if it is afterward found that the grades or elevations were in error. It is understood, of course, that this statement is not applicable in a case that involves deception. The author

has called attention to the rule that no man may contract against his own fraud, City of Lima, Ohio *vs.* Farley (11), Bridger *vs.* Goldsmith (12).

Prior Work on Same Site.—The question sometimes arises as to the right of the contractor to rely upon information obtained from prior contracts. Dunn *vs.* City of New York (23) shows that a bidder, who looks at a prior contract and assumes without investigation that the work required by that contract has been fully performed, does so at his own risk. In that case a paving contractor assumed that the grading had been done under a prior contract. His own contract required that he do grading and he could easily have ascertained whether or not any grading was necessary. The Court held that the specifications of the prior contract, which were not made a part of his contract, did not constitute a representation on which he could rely.

In Cauldwell-Wingate Company *vs.* State of New York (35) the specifications for the prior contract were referred to in the claimant's contract. The contract was one to build the superstructure of the State Office Building in New York City. The prior contract was one to build the foundations. Because of unexpected sub-surface conditions, additional work requiring nearly a year for its performance was required of the foundation contractor, and this delay increased the cost of the superstructure. The contractor for the superstructure recovered, notwithstanding the contract provision that he should not be entitled to damages for any delay from any cause, including the act or neglect of the State.

Cases in Which There Was No Representation.—To establish a claim for increased compensation on account of an alleged breach of warranty the contractor must show that information has been given to him upon which he relied in making his bid and that such information was erroneous. He must show also that he had a right to rely on the information. The claims of the contractors in several of the cases cited by Mr. Herwitz failed because no representation had been made.

An example of this is Allen N. Spooner and Son Inc. *vs.* City of New York (21) in which the contractor complained that he had encountered loose rock in the construction of a pier. The City had shown two typical cross-sections, one for solid rock and one for loose rock, and had stated in the contract that no core borings had been made.

In another case, Maryland Casualty Company *vs.* Board of Water Commissioners (39), rock grades were shown on the plan but specifications required that the excavation be carried to the grade shown or to such increased depths as may be necessary to obtain a firm foundation. The contract contained a separate unit price for extra excavation below the grade shown, and this price had been paid to the contractor. The Court ruled that this negatived the idea of a representation that the rock would be at the grade shown, and that the contractor had received all of the compensation to which he was entitled.

In Simpson *vs.* United States (41) the work was done according to plans which the contractor had himself prepared without any information whatever from the United States as to sub-surface conditions. It was his responsibility,

not that of the United States, that the design was not suited to the conditions which existed on the site.

Weston *vs.* State of New York (63) was a case in which the contractor claimed increased compensation because he was required to excavate more rock than was indicated in the engineer's estimate. He had been paid the rock-excavation price for all rock removed. The Court ruled that as there was no claim of deception, inequality, or inequity, the contractor was not entitled to increased compensation.

The Weston case (63) presents the question of the contractor's rights under a unit-price contract where there has been an increase in the quantity of one item. In the usual case such increase does not entitle the contractor to any payment beyond the unit price bid; but this rule is not applied if the increase is so great as to change, radically, the nature and cost of the contract as a whole.

Specifications Should Describe Finished Work, Not Methods of Construction.—The aforementioned cases have dealt with representations as to the work to be done or as to conditions affecting the work. There is another group of cases that should be helpful to engineers in the matter of drafting specifications. A contractor who has done the work according to the specifications is entitled to be paid the contract price although the result may not be what the owner anticipated. In three of the cases cited by Mr. Herwitz, MacKnight Flintic Stone Company *vs.* Mayor (65), Filbert *vs.* Philadelphia (65), and Kuhs *vs.* Flower City Tissue Mills Company (48), the owner sought to obtain water-tight construction, and for this purpose gave to the contractors specifications which described in detail how the work should be done. In each case the contractor had done the work according to the specifications, and in each case the Court ruled that the contractor was entitled to be paid even if the structure leaked. In another case, Montrose Contracting Company, Inc., *vs.* County of Westchester (66), the owner had specified the methods to be used in the construction of a sewer tunnel. It was shown that those methods were appropriate for the construction of a tunnel in free air, but not for one in compressed air, and the Court held that the specification of those methods constituted a representation that the job was a free-air tunnel.

Mr. Herwitz has presented two cases which involved the same legal principle as the foregoing cases, but which relate to the plans rather than to the specifications. In United Construction Company *vs.* Town of Haverhill (38) the Court held that a contractor who had built bridge piers to solid ledge, at the depths indicated on the drawings, was entitled to be paid even if the piers were washed out in a subsequent storm. In United States *vs.* Spearin (65) the Court held that a contractor who had constructed a drain and had connected it with an existing drain, as required by the plans, was entitled to the damages suffered by reason of the fact that the existing drain was not sufficient to carry off the water which accumulated on the work during a storm.

The principle that underlies all of the cases in this group is that the owner is responsible for the sufficiency of the drawings and specifications. If, because of defects in the drawings or specifications, the completed work is defective, or the expense of doing the work has been increased, or the contractor has been

induced to bid too low a price for the work, the responsibility is upon the owner, not upon the contractor.

Questions Arising During Performance of Work.—Mr. Herwitz has also cited a group of cases involving questions that have arisen during the performance of contract work. The questions presented do not relate to the assumption of risk of subsoil conditions, which is the subject of Mr. Herwitz's paper, but they are of such importance to engineers and contractors that it is fortunate that he has considered them.

The first of these questions relates to the engineer's final certificate. Nearly every construction contract contains provisions that the final certificate of the engineer is conclusive upon the parties and that such certificate must be made and filed before the contractor is entitled to receive his final payment. There is no doubt that if the certificate is made in good faith and in accordance with the terms of the contract, it is conclusive, and the contractor cannot recover as payment under the contract more than the sum certified, O'Brien *vs.* City of New York (5) and United Construction Company *vs.* St. Louis (32); but, if the Courts are convinced that an injustice has been done either because the engineer has been arbitrary in the making of his final certificate or because he has misconstrued the contract, they do not hesitate to set aside his certificate and to determine for themselves the sum due the contractor, White *vs.* United States (77), L. P. & J. A. Smith *vs.* United States (78), St. George Contracting Company *vs.* City of New York (79). The rule is firmly established that the final certificate is not a bar to an action for damages for breach of contract or breach of warranty in the making of a contract, Kuhs *vs.* Flower City Tissue Mills Company (48), Faber *vs.* City of New York (22), and Borough Constructor Company *vs.* City of New York (41).

Changes in Work.—Few large jobs are completed without changes in the drawings or specifications, additions to the work, or elimination of parts of the work. If the work is rendered more expensive, whether because of unforeseen conditions existing when the contract was let, as in Faber *vs.* City of New York (22), or because of any act or neglect of the owner after the making of the contract, as in Sundstrom *vs.* State of New York (9), Feeney and Sheehan Building Company *vs.* State of New York (9), Thileman *vs.* City of New York (55), the contractor is entitled to increased compensation. In such case no written order is necessary and the sum recovered by the contractor is technically recovered as damages, not as payment for extra work.

The elimination of work frequently results in hardship to the contractor because in making up his bid he has apportioned his indirect cost, overhead, and profit over the pay items of the contract. The elimination of work does not usually reduce the amount of the contractor's indirect cost and overhead, and it certainly deprives him of a part of his undistributed profit. Notwithstanding the hardship imposed, the Courts do enforce contract provisions permitting the elimination of work without liability for anticipated profits. Kinser Construction Company *vs.* State of New York (64), and in the case of New York City subway contracts, the Courts have enforced a provision that work may be eliminated without liability to the contractor for anticipated profits, overhead, or damages. In Del Balso Construction Corporation *vs.*

City of New York (64) the Court said that the contractor must have had the contract provision permitting omissions in mind in calculating his overhead when preparing his bid. Even if the bidder does have it in mind, however, what can he do about it? He cannot foretell what items will be omitted, and certainly the City will not want bidders to add a contingent item to cover possible omissions of work. In view of this decision, the Board of Transportation, City of New York, should revise its omission clause, or, if it wishes to retain the unrestricted right to omit parts of the work, it should make provision for payment to contractors of the proportionate part of indirect cost and overhead applicable to the work omitted.

Conclusion.—A discussion of these cases would not be complete without a restatement of the rules to be observed by the engineer, and other rules that should be observed by the contractor. Among those rules the following stand out as most important:

For the engineer—

(1) The "Description of Work" in a proposed contract must describe, correctly, all of the work that the contractor will be required to do;

(2) All available sub-surface information should be given to bidders with complete information as to how, when, and by whom such information was obtained;

(3) A general statement concerning sub-surface conditions should not be made unless it is known that such statement is correct;

(4) An owner who has given the contractor inaccurate or incomplete boring information is responsible even if the boring information is not made part of the contract;

(5) A contractor is not required to make surveys, soundings, or borings to determine whether or not the information given him in the contract is correct;

(6) The Courts do not look with favor upon a one-sided contract, a contract that attempts to shift all of the risk to the contractor;

(7) If there is a subject as to which information is not complete and as to which it is intended that the contractor shall assume the risk, the contract should refer particularly to that question and the risk to be assumed should be clearly stated; and

(8) The specifications should tell a contractor what he is to do; they should not tell him how to do it.

For the contractor—

(9) The entire contract and specifications should be studied with particular reference to any provision directing attention to a special subject (it may be assumed that the engineer did not insert such provision unless the subject was one concerning which there was some uncertainty);

(10) A bidder should always inspect the site and is charged with knowledge of everything visible on such inspection;

(11) A bidder is not usually required to make borings or other sub-surface investigation, particularly if that work has been done by the owner; but the bidder is required to make any reasonable investigation which time and circumstances permit;

(12) Unless a prior contract for work on the site is referred to in the contract for which a bid is being made, the bidder is not justified in assuming that the work has been done according to the drawings and specifications of such prior contract; and,

(13) The owner is primarily responsible for the sufficiency of the drawings and specifications; but this will not protect the contractor if he knows, from facts presented to him or facts that were visible when he made the contract, that the drawings and specifications were not adequate.

Court Citations and References.—

(77) White *vs.* United States, 241 U. S. 149.

(78) L. P. & J. A. Smith *vs.* United States, 256 U. S.

(79) St. George Contracting Co. *vs.* City of New York, 205 N. Y. 121.

David A. Molitor,[7] M. Am. Soc. C. E. (by letter).[7a]—Every sizable construction contract involves at least some features, the details of which are not predictable at the time of letting, and these must be appraised, when encountered, and allowed for in the final settlement.

Despite the most careful soil investigation and borings, made to obtain data for foundation design, there will usually develop, during construction, more or less material variations in the character and required depth of excavation, resulting in modifications from the design as originally prepared. To pass this risk on to the contractor, as is usually done, by introducing cleverly worded clauses into the specifications, is unwise and unbusinesslike, and generally leads to a case of arbitration or lawsuit.

If the designing engineer expects to deal honestly with the prospective contractor, he should write his specifications with that end in view. He should expect to pay for everything that is stipulated to be done or furnished without hoping to get something for nothing. Neither should the owner have to pay for anything not obtainable in fact.

Therefore, why not write a clearly worded specification covering everything, so far as knowable, in exact businesslike language, leaving the unforeseen eventualities to be dealt with under supplemental written agreements, and thus avoid serious disputes at the time of final settlement?

The wide variety of cases cited by the author illustrates the attitude of the Courts when called upon to clarify the business relations between owner and contractor, resulting from doubtful or contradictory requirements in the specifications with regard to the amount of work to be performed, usually under a lump-sum contract.

If the engineer cannot stipulate, clearly, the exact quantities of work which the contractor is to perform, and yet expects to hold the latter to a lump-sum price, he must also expect to obtain excessive bids covering the unreasonable, or inadequately described, features of the proposed work. Later, if the contractor finds himself a party to a losing contract, the case goes to Court for the final adjudication of disputes following in the wake of an ambiguous contract. As a matter of fact, lump-sum bids are generally unfair to both owners and contractors and should be avoided.

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[7] Cons. Engr., Harlingen, Tex.

[7a] Received by the Secretary February 14, 1939.

The practice of awarding contracts only to the lowest responsible bidder makes it mandatory to provide a method of obtaining comparable bids, but not necessarily lump-sum bids. This can best be accomplished by incorporating in the tender an approximate estimate of quantities of each kind of work or materials to be furnished, the bidder merely supplying the requisite unit prices to determine the aggregate amount of the proposal. The same unit prices should later apply to changes in the estimated quantities made necessary during prosecution of the work by additions to, or abatements from, the estimated quantities in arriving at the final estimate and settlement.

Extras, limited to work or materials not covered by the drawings and specifications, and hence not contemplated under the contract, are to be paid only in accordance with supplemental written agreements on a cost-plus basis.

There will always be abundant evidence to cast doubt on the most carefully made soil investigation, and nothing further is necessary to start a controversy whenever the contractor sees his way clear to recover. One of the most fertile contentions for extra compensation is that not all the information was disclosed at the time of bidding.

The result obtained from the soil investigation should be accessible to all bidders merely for what it is worth, frankly recognizing the approximate, or even erroneous, character of the information, with a clear statement that all departures from the design data and changes in foundations as finally decided upon and built, shall be adjusted in the final estimate at the unit prices named in the bid.

If each intending bidder were to obtain his own information, this would necessitate duplicating the soil investigation, which is impractical. Any requirement whereby the bidder is to obtain technical data relative to depth and character of foundation, ground-water, pumping, sheet-piling, etc., and then be held responsible for the outcome, is manifestly unfair as it amounts to doing precisely what the designing engineer is supposed to do to obtain the required information for his own use and for all intending bidders. Contractors should not be penalized for shortcomings in the design. This is the responsibility of the engineer.

The very valuable advice contained in Manual No. 8 may be regarded as an enabling act to specification writing; but here again one finds a wide divergence in the quality of the product put out by different writers. It is sometimes easier to tell the other fellow how to do something than to do it oneself.

The author has presented innumerable cases which terminated in disputes in connection with "unexpected risks," and has cited the decisions rendered by the Courts on the basis of facts disclosed in each case. Although this information is very interesting and instructive, the matter of avoiding a recurrence of such cases is still an important issue. The writer contends that there is no justification for writing a contract based on a gamble.

F. B. Marsh,[8] M. Am. Soc. C. E. (by letter).[8a]—The conclusion that "a knowledge of what the Courts have done in similar circumstances lessens the

[8] Div. Engr., Eastern Dept., New York Board of Water Supply, White Plains, N. Y.

[8a] Received by the Secretary February 18, 1939.

element of uncertainty for the future" (see "Conclusion") is justified only for those types of cases in which the Courts agree among themselves. However, a reading of this paper confirms the previous feeling that in too many cases one Court decides in favor of the plaintiff and another Court, with a quite similar set of circumstances, decides the other way. One cannot escape the feeling that in one case the plaintiff's counsel or witnesses have appealed successfully to the Court's sympathy, or perhaps the weather or some other extraneous matter has given the Court a more generous disposition. Certainly the evidence alone does not account for many rulings, and precedents can be found for almost anything.

The one important fact that stands out is the obligation resting on the engineer to show the available sub-surface information not only accurately, but completely, including, for example, such data as the records of borings that had to be abandoned before the desired depth was reached. It may be that the reason for abandonment may, in itself, give a clue to sub-surface conditions not fully revealed by a subsequent deeper hole near-by. In any case, the listing of abandoned holes with the others is obvious evidence of the engineer's effort to make all information available.

Evan S. Martin,[9] Assoc. M. Am. Soc. C. E. (by letter).[9a]—The author views the problem and record from the legal standpoint. It has been common to insert, in construction contracts, general clauses that transfer the risks from the owner to the contractor. Acceptance of many of these risks may be optional with the owner, or he may pass them along; but certainly it is the duty of the owner to define the substance of the contract—to describe to the contractor what he is to do—if he invites a bid price.

The paper is predicated on the assumption that, in sub-surface construction, there are facts, essential to the completion of the contract, that have been overlooked in preparing plans and specifications. The writer maintains that a clause which places all risk of sub-surface conditions on the contractor renders the contract, basically, null and void. To cite an above-surface parallel: A contractor cannot be enjoined to construct a 12-room residence "of the best material and workmanship, to the satisfaction of the owner" without plans and specifications—that is, without knowledge of what is above the surface. The lawyers and Courts will not say so; they have not said so yet; but this continent has not emerged mentally and morally from wild free-shooting frontier conditions. When the people do emerge and really establish law and order, they shall be civilized because they will consider equity first and only. Whether the owner pays less for investigation than he does for contracting for something "in the dark" is irrelevant.

When the Construction Division was organized, in 1925, the writer stated that he thought that this section should consider the development of rules for the interpretation of contracts—an engineers' jurisprudence. There is no reason for engineers to defer to the legal profession in defining the ethics of their practice. Note the conflicting opinions cited in this paper; they are not much

[9] Secy.-Treas., Jas. A. Wickett, Ltd., Toronto, Ont., Canada.

[9a] Received by the Secretary March 6, 1939.

concerned with what is fair but, as a lawyer might say, what is fair "in the light of the contract." The writer's point is that the "light" is not in the contract but in equity behind the contract. What is fair is simple, in interpreting contracts; it is that neither party shall get something for nothing. In legal parlance "something for nothing" is "without consideration," and therefore null and void. It is not fair to force a contractor into the surety business; he is not organized to undertake risks, and he cannot know what to charge for them.

Engineers (and Courts, also) must delve behind the contract, "the law," and "the Constitution" to what is fair. Stated inversely, engineers must make ethics—equity—the sole guide in the interpretation and adjudication of contracts. Perhaps this is what is intended by Blackstone's British Common Law.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

FLOOD-PROTECTION DATA

PROGRESS REPORT OF THE COMMITTEE

Discussion

BY MESSRS. EDGAR E. FOSTER, AND C. D. CURRAN

EDGAR E. FOSTER,[5] ASSOC. M. AM. SOC. C. E. (by letter).[5a]—The effort spent in searching for and analyzing data of historic floods is well spent. Because of the short records usually available, all records of high floods are valuable wherever found. This statement is particularly applicable in the case of the truly great floods so rare that they occur only once in several generations. These data, like all historical knowledge, form a background so that a better understanding can be obtained from the shorter but complete records of discharge.

Nevertheless, historical records of isolated floods cannot replace any part of a complete record. The value of such historical data is limited in two ways: First, each item is an isolated value and, as such, cannot be utilized to advantage as it could be if it were in a continuous series; and secondly, as the Committee implied in one statement, the relationship between stage and discharge has been altered profoundly by economic development along streams so that a given historical stage has little correlation with present stages and determination of early discharge is uncertain. This statement assumes that the historical stage can be fixed within a reasonable degree of accuracy; but, quite frequently, it cannot be known definitely. At best, these early flood heights are known only in the long-settled communities.

The change in the relationship of stage and discharge must be regarded as a serious defect in the use of records antedating continuous observations. The change is probably always one to increase the stage for a given discharge because of the continual encroachment on the flood-plain, and even on the channel, by works of Man. In the Pittsburgh example cited in the Committee report, the discharge might have been as great or greater in 1763 than in 1936. An ex-

NOTE.—The Progress Report of the Committee on Flood Protection Data was presented at the Annual Meeting, New York, N. Y., on January 18, 1938, and published in January, 1939, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the report.

[5] Associate Engr., U. S. Engr. Office, Washington, D. C.

[5a] Received by the Secretary February 17, 1939.

ample of the effect of waterways encroachment is found in the record on the Potomac River at Cumberland, Md. There is a record of a flood in 1889 with a stage of 13.3 ft (U. S. Weather Bureau). The U. S. Geological Survey estimated the discharge at a station 2 miles down stream to be 88 800 cu ft per sec. The flood of March 17, 1936, had a stage at Cumberland of 17.65 ft and a discharge of 88 200 cu ft per sec. With practically the same discharge there was a stage in the city which was 4.35 ft greater in 1936 than in 1889, which increase could have been caused only by encroachment on the channel. This encroachment, like that in any other locality, increased as the community developed; but it was not necessarily uniform or continuous. This irregularity, unknown in quantity and time of occurrence, precludes any reliable adjustment of discharge, and consequently, decreases the utility of historical floods in present-day problems.

The Committee discussed the use of statistical methods in the analysis of flood problems and apparently reached the conclusion that such methods are not applicable. This conclusion is at considerable variance with the experience of the writer who has used those methods with what he believes to be considerable success. Manifestly, statistical methods do not furnish all the answers to questions of floods because those problems are too diverse to be solved by any one line of investigation. For certain phases of flood studies, however, the writer believes that they are indispensable.

Before discussing the phases for which statistical methods may be used, it is desirable to dispose of one subject for which they are not suitable. That subject is the determination of a possible maximum (not probable maximum) flood. Statistics reveal what has happened; if sufficient observations are available, they will indicate what will probably happen; but, they cannot tell all that can occur unless the maximum has already been experienced. Statistics reveal the maximum flood of record, but the record does not necessarily include the maximum possible flood. That flood can be estimated best by meteorological and hydrological studies to determine the greatest storm run-off, constructing the flood by means of the unit hydrograph.

The principal purpose of a frequency curve is the calculation of annual flood losses for economic study and comparison with costs of proposed protection; and for that purpose the writer has found the statistical methods (meaning the methods based on the mathematical theory of statistics and the theory of probability) by far the soundest and most satisfactory.

The Committee states in Paragraph 9(*a*) that a fundamental error is involved in the assumption that all flood data are homogeneous. Is this assumption really an error, fundamental or otherwise? Assuredly, a series of floods can be selected so that each will be substantially independent of the others, barring the accidental or chance occurrence of one flood on the end of another immediately preceding, and sufficiently large so that ground-water or, rather, its fluctuations, are negligible as compared with the total discharge. The diversity of magnitude constitutes the sole feature of variation and is the variable of the distribution function. Such a series is composed of data of flood magnitudes and is certainly as homogeneous in nature as any series can be.

Farther along in the same paragraph the Committee reveals that it refers to homogeneity of causes of floods and not that of data; but, diversity of cause cannot be admitted as a defect of the series of data, provided chance is the controlling element of magnitude and occurrence. Assume for the moment that the causes of floods given by the Committee—namely, "tropical hurricanes, by cloudbursts, by rapid snow melting plus rain"—are wholly unrelated phenomena. This condition implies that the observed floods arising from one cause are independent of those resulting from another. Independence of observations is one of the basic requirements for the application of the methods of statistics and probability so that the condition which the Committee alleges is an obstacle to the use of statistical methods is in reality an additional reason for their use.

A similar situation of independent causes arises in the theory of errors. As stated by the late Mansfield Merriman,[6] M. Am. Soc. C. E., an actual error in reading a level-rod, for example, is the net result arising from different causes, many or all of which may be wholly independent. Nevertheless, no one questions the applicability of the normal or Gaussian probability function to the solution of the problems of errors.

It is true, of course, that the Gaussian function is not applicable to extreme skew data such as flood magnitudes; but, it has been shown[7] that the Gaussian or normal equation is only a small part of the theory of probability. That theory has been expanded and other probability functions have been developed. One such function contains a logarithmic transformation of the variate; that is, it uses the logarithms of the errors instead of the errors themselves as a variable. This type will handle the data of flood magnitudes as readily as the Gaussian handles the data of errors.

It is a question whether or not anything could be gained by dealing separately with the floods resulting from various causes. The total number of floods experienced in a given period of time—say ten years—is the sum of those resulting from the various individual causes; likewise, the probability of the occurrence of one flood from any cause is the sum of the probabilities of a flood from the individual causes. Furthermore, in dealing with floods of any one particular location, empirical constants (parameters, or semi-invariants) must be derived in the statistical methods from the observed data as for other methods. (In the statistical methods the constants are tied into sound mathematical theory which is not done in other methods.) It does not seem probable that any advantage can be gained by separating the floods in accordance with the causative factors over the results to be obtained by combining the floods of all causes. On the other hand, there will be fewer data for each flood cause and a consequent diminution of accuracy of the derived constants.

It may not be out of order to discuss here some of the causes of floods and their variations. Floods are the result of two general causes—excessive rainfall and rapidly melting snow. These causes may operate singly or in combination. The precipitation is brought in various types of storms, including cloudbursts and hurricanes.

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[6] "Methods of Least Squares," by Mansfield Merriman, pp. 17 and 18.

[7] "Probability and Its Engineering Uses," by Thornton Fry.

In a fixed locality on a given stream, floods will result from favorable storms and other conditions prevailing in the region. For the larger water-sheds in the temperate zones, floods are the result of the rapid lifting of a warm, moist, tropical air mass over a mass of cold air, an action not infrequently operating under an unusually steep gradient of barometric pressure. The occurrence of such lifting over a particular basin is a matter of chance, depending upon the variable meteorological factors causing the storm. It may occur over the center of the basin which, in that case, would receive the maximum precipitation; or, it may occur in an adjacent region so that only a fraction of the maximum would be felt. All elements of the storm are subject to variation: Humidity or moisture content of the air, steepness of slope of the cold front and the consequent rate of lifting, wind velocity (which is the rate of supply of moist air), and finally, the precipitation, which varies from place to place and time to time. However, not every contact of diverse air masses results in excessive precipitation. Likewise, the area covered by heavy rainfall varies widely from small areas covered by cloudbursts to 1 000 or more square miles covered by extensive air masses.

The snow cover and its water content varies greatly since it is dependent upon the precipitation of the preceding season and the occurrence or absence of previous snow melting. The existence of a heavy snow cover at the time of arrival of a warm, tropical air mass is a chance contingency; and, finally, the condition of the ground surface varies much from season to season, and may be a sufficient factor to make or prevent a flood.

In fact, variation in magnitude and in other characteristics is a conspicuous feature of all meterological and hydrological phenomena. It is only natural, therefore, that the mathematical theory of probability as utilized in the statistical methods should be the logical method of analyzing flood data.

The probability of many meteorological phenomena is calculable by well-known laws of probability. The distribution of the magnitude of annual rainfall fits the normal law[8] closely. Studies made by the writer show that the distribution of the annual number of thunderstorms (of which the cloudburst is probably a variant) and the tropical hurricane follows closely the Poisson law of small numbers, namely,

$$P(n) = \frac{e^m \, m^n}{n!} \qquad \text{(1)}$$

in which $P(n)$ is the probability that n events (thunderstorms or hurricanes) will occur in a given interval of time when m is the average number and e is the base of the natural logarithms. The probability of magnitudes of daily rainfall may be calculated as a logarithmic transformation of the normal probability function. In view of these considerations it seems to be inevitable that floods which are only a result of meteorological and hydrological causes should be amenable to the theory of statistics. This does not mean, however, that any statistician can compute the frequency of floods by formula, but it does mean that the engineer trained in hydrology has another powerful tool for his use.

[8] "Study of Variations in Annual Rainfall of Oahu," by W. T. Nakamura, *Monthly Weather Review*, Vol. 61, p. 354.

It is true that the statistical methods do not permit much consideration of the historical floods because the basis of such methods rests on a complete distribution of magnitudes, from the lowest suitable value to the highest. This distribution of small, medium, and high floods is of such importance, however, that the omission of a few scattered historical floods from the computation is a relatively small matter.

The Committee (see Paragraph 9(*c*)) seems to emphasize the factor of judgment. Other viewpoints seem preferable to the writer. Mr. Charles O. Hardy, in discussing economic risk, declares[9] that estimates of probability fall into three classes—(1) mathematical, (2) statistical, and (3) judgment. He states further that judgment is only a crude application of the statistical method. Usually judgment is used unaided only when nothing is available to serve as a basis for calculation. However, every suitable aid should certainly be used in its guidance.

Nevertheless, the statistical method leaves ample opportunity for the exercise of pure judgment on the part of the hydrologist. An opportunity presents itself at every application of the theory to a set of observations. It may be desirable, for example, to weight certain observations. Judgment is indispensable in the interpretation of the results; but this exercise of judgment is required not because judgment is superior to calculation, but because no calculation is possible in certain imponderable phases of the work.

As the records of stream flow become longer, the data of flood discharges become more plentiful; the distribution curves (that is, in the statistical sense, the proportion of data of low, medium, and high magnitudes) become better defined. The increase in data of high floods especially helps to determine the probability of extreme floods. It is difficult to understand why longer records will affect unfavorably the use of statistical methods of analyzing flood data any more than the approximate graphic methods. However, the increased records obtained as time passes will render the data of isolated historical floods gradually less valuable because the additional continuous data of flow will be more accurate. Furthermore, the latter data will be less affected by the early development of the country and the stage-discharge relationship will be more certain.

It is difficult to understand why the Committee should reject the statistical methods of analysis of flood data. Progress in engineering as in other fields of endeavor means change to better methods; it means sounder methods of analysis and more accurate and dependable results. The statistical methods in the hands of an engineer trained in hydrology will constitute a tool to achieve such results. The key to understanding the attitude may lie in the phrase, "applying the methods of least squares to natural phenomena***." The methods of least squares are based entirely on the Gaussian or normal probability function which constitutes only a small part of the theory of probability used in statistics. The normal equation is limited to few fields of application, among which the analysis of errors is a conspicuously successful example. Other methods and other probability functions are required for the extremely skew

[9] "Risk and Risk-bearing," by Charles O. Hardy, pp. 42 and 43.

functions of flood data. Since such functions have been developed[7, 10] there is no reason to reject statistical methods because the theory of least squares cannot be utilized.

With respect to the subject of flood damage, the Committee is correct in stating that research is needed to determine what may be considered true flood damage and to determine the relation between flood loss and what expenditures are justified for flood protection. Such research should undoubtedly be fearless and unbiased. It should be just both to those (in most cases, taxpayers) who will pay the cost and to those who have borne the burden of the economic development of the country.

Damage by floods may be compared to damage by other fortuitous causes or events such as fire, tornado, or earthquake. The loss of structures that survive ordinary floods and fail in unusually large ones should surely be accepted as true flood loss. To ascribe such losses to an intangible factor like "human error" is to require a foresight in the builders equal to the hindsight of the Committee. A house may be constructed of inflammable material, but if it is burned, it is a fire loss; a structure may be flimsily constructed, but if it is destroyed by a tornado or hurricane, it is still chargeable to those elements. Likewise, if a dam stands up in ordinary freshets, but fails in a large flood, it is still a true flood loss. It is indeed fortunate that the bridges over the Ohio River did not fail during the flood of January, 1937; but, if they had gone out, the damage could still be considered a true flood loss which would equal the cost of replacement less depreciation for the life of the structure.

Probably all cities and at least the large towns were founded on their sites because economic factors favored the location. Unless such factors promoted their growth, the first settlements would not have developed into the existing large communities, and as long as those factors continue to operate, it is practically certain that the cities will remain in spite of menace from floods. For example, the city of New Orleans, La., was settled and grew into a large city because of its location, in spite of floods. Likewise, cities in New England were built on the banks of streams to obtain water power, regardless of flood menace.

Once a large community has developed in a location, it is extremely difficult to move it, for such evacuation means the abandonment of much of the accumulated wealth. People will adhere to the site where their wealth is invested. It is the problem of the engineer, therefore, to provide such protection as may be economically justifiable for the existing communities. Mass evacuation appears possible only in the case of the smallest settlements and scattered fringes of the larger cities.

The question as to whether one factory or a small community should be moved to another location is answered by the same means—that is, by study of economics—provided danger to human life is not an issue. The question to be answered is simply whether it is cheaper to pay the recurring flood loss or to pay the cost of moving. Perhaps a large industrial plant can stand the loss caused by infrequent floods more easily than it can the high cost of evacuation. However, some form of flood protection will very likely prove to be a better answer.

[10] "Mathematical Theory of Probabilities," by Arne Fisher; and "An Asymmetric Probability Function," by J. J. Slade, Jr., *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 35.

The Committee is correct in its statement that certain classes of structures (Class III) should be constructed outside of flood zones or be made flood proof. Those structures house institutions whose destruction would vitally affect the lives and safety of human beings.

C. D. CURRAN,[11] ASSOC. M. AM. SOC. C. E. (by letter).[11a]—The progress made during the past three years, in compilation and publication of flood data, and in co-operative research into the hydrometeorological fields, is a cause for commendation to all concerned and to the Committee in particular for its directive thought and effort.

In the move to publish data and make them available to the profession at large, care must be exercised to avoid over-production. A decade hence too many repetitious publications can be almost as detrimental and confusing as too few available data were a few years ago.

The Committee does well to caution against blind use of statistical methods. Forecasting future flood magnitudes by probability methods is likely to be highly inaccurate. An understanding of the limitations of probability theories is essential. The assignment of a time frequency to a flood stage or magnitude is often misleading, at least to the laity, who are vitally interested in floods and flood protection. The term "100-yr flood" is concise but not precise. It would be better to refer to it as the "1% chance flood." It would be even better to describe the flood by use of a subsequent clause rather than an antecedent phrase; for example, "that flood which it is at present estimated will be equaled or exceeded during 1 yr in a hundred." In many instances the entire idea of frequency can well be omitted without detriment to the study or context.

Nevertheless, statistical methods have their place in statistical problems. Economic studies to determine the worthiness of flood protection projects lend themselves well to probability methods. Results may need frequent revision in the light of new records, but the best approximation to the answer sought can be obtained by use of flood and damage frequency studies. It might be well to limit the use of probability studies to this phase of the problem.

The Committee is correct in stating that flood damage data are next in importance to flood records in basic consideration of flood protection. It is not meet in this comment to present a thesis on the mechanics of compilation and classification of damage data. However, in passing, it is perhaps proper to note that many compilations made within recent years are of dubious value in determining the economic worth of protective measures because their groupings are by political divisions rather than by stream basins.

A basic philosophy and policy on flood damages must be established or there can be nothing except confusion of thought. The definition of losses may be limited to matter destroyed and rendered unfit for its intended usage and to energy uselessly dissipated. On the other hand, flood damages may be considered to include every economic and social disturbance that results either directly or indirectly as a result of a flood.

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[11a] Received by the Secretary March 20, 1939.

The Committee sets forth three classes "of damage that does not appear to be properly chargeable to floods." The context indicates that "chargeable" is synonymous with "attributable." In considering flood damage, present development must be considered correlatively. If the damage would not have occurred at the particular time it did occur without the existence of a flood, then the damage is a flood damage. It is true that certain damages should not be considered when computation of prospective benefits from protection is made because certain damages are non-recurrent or non-preventable; but this fact does not make the damages other than flood damages.

If the thought in the report were carried to its ultimate it can be held that no damages are chargeable to floods. All damages from floods are chargeable to inadequate planning and lack of foresight. This approach becomes academic, however; a practical solution is needed. The problem is to ascertain what damage elimination may properly be considered a benefit. A corollary problem then is to determine against what items benefits should be charged.

Damages to the structures of Class I are probably non-recurrent except for that part that might be considered an increased maintenance cost. As the Committee states, the structures should be proof against floods. Repair of flood damage would then be a proper maintenance charge. However, present structures not proof against floods are in place. Damage to them cannot be ignored as not chargeable to floods. The consideration should be whether future damage can more economically be cared for by flood protection works or by local maintenance. If a bridge is destroyed, its value is not a proper item to include in prospective benefit computations because its replacement should be proof against floods. However, if it is not destroyed and is not proof against probable future floods, then benefits to be obtained by its protection are a proper consideration.

Items of Class II may be considered similarly. Damages are more or less non-preventable by flood protection works. However, if flood protection works will lower maintenance costs, the benefits are properly included in determining the economic worth of the proposed project.

Perhaps it is only with the language that the writer disagrees and not with the basic thought of the Committee. He considers all damages by floods as attributable to floods, but believes that in the determination of prospective benefits from protective works certain damages should not be considered. This is apparently in agreement with the comments of the Committee on its discussion of Class I and Class II items.

Items of Class III certainly should be considered in ascertaining prospective benefits. The structures of this group should not have been built in the flood plain, but, since they are so located, they must be protected. Actual and possible flood damages should be considered to determine whether flood protection to the development, as it exists and will probably be expanded, is economically sound. Future development can be regulated to prevent damage to the new improvements, but present developments must be considered in their present status. Economic consideration of flood protection should not be based on a determination of whether it would be justified at present if

those structures which should have been made proof against floods had been so built.

It is certain that damage data must receive careful study lest benefits include duplication, improper credits, or inflated values. Restoration of property depreciation values and appreciation of property are considered by some to be benefits. These may too readily be exaggerated. The writer had occasion within the past year to investigate these items in a community. It was found that manufacturers hesitated to buy certain lands subject to recent floods. Accordingly, the land value was said to have depreciated. Yet in the same community, market values of occupied land, equally subject to flood, had not fallen. The lack of logic there indicates that investigation must be careful. Neither realtors, tax assessors, nor Chambers of Commerce desire to recognize such a loss if it exists. Limited supply and high demand would possibly prevent depreciation in values because of flood, whereas surplus supply and limited demand would tend to exaggerate depreciation.

Furthermore, in the determination of this class of flood-protection benefit there is great possibility of overlap with direct damage-reduction benefits. Direct damage may be considered a loss due to an increase in rate of depreciation or cost of maintenance. As such it is immediately reflected in a drop in value of the property. Both are measures of the same loss. However, property depreciation would probably also include a natural evaluation of intangibles. It would appear, therefore, that in the determination of potential benefits the direct damage to private real property should not be considered if the value of property depreciation can be found. Consideration must also be given to the fact that the former is in the nature of an income loss whereas the latter is a capital loss.

An example similar to one cited by the Committee has been noted. A manufacturing plant selected a low flood-plain area at a river bend as the site of development. In making the selection flood hazard was recognized but probable damage was offset by decreased cost in pumping water for manufacturing processes. After heavy losses in a recent flood, the concern, stating its damages and for the time being omitting its benefits from years of low-head pumping, sought public aid; and, rather interesting, although irrelevant here, was the fact that the concern was financed primarily by foreign capital.

Although the writer disagrees with a part of the Committee's discussion on damages, he concurs with its principles and can only urge that the work be continued to the end that eventually adequate and proper data are soundly used.

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DISCUSSIONS

BEACH EROSION STUDIES

Discussion

By Morris N. Lipp, M. Am. Soc. C. E.

Morris N. Lipp,[4] M. Am. Soc. C. E. (by letter).[4a]—The mention, in this discussion, of certain phases of the beach erosion problem which Colonel Brown did not set forth in his comprehensive paper, is not intended as a criticism of his failure to discuss these matters. They are presented in accordance with his request that his paper be the subject of full and free discussion by all interested engineers.

With full realization of the correctness of his assertion that every locality presents a different problem, a discussion should essentially be confined within the limits of an engineer's own experiences and observations, in order that an accumulation of discussions will establish some definite fundamentals. It is first necessary to know why certain problems exist, whether they are Man made or Nature made. If caused by the activities of Man, the engineer in charge of such activities should be fully cognizant of the future problems that are created.

Two of the factors that contribute to the problem of beach protection are the desire to erect buildings on ocean-front property, and the construction of inlets for sanitation and navigation. It is not within the province of the civil engineer to protest these desirable and necessary activities when kept within reasonable limits, but it is his duty to help solve the problems they produce.

Colonel Brown cites the necessity for geological studies and establishes vital points in his discussion of littoral drift. Once the direction of drift is determined an important barrier is hurdled. It is now accepted as true that the general movement of sand along the Florida east coast is in a southerly direction. This principle was established by the Florida State Geological Division. Analyzing sand at various points along the coast, and tracing the sand to

Note.—This paper by Earl I. Brown, M. Am. Soc. C. E., was presented at the meeting of the Waterways Division at Jacksonville, Fla., on April 21, 1938, and published in January, 1939, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[4] City Engr., Miami Beach, Fla.

[4a] Received by the Secretary March 3, 1939.

its source of supply, entered into its studies. The engineer, as Colonel Brown points out, sees evidence of the direction of drift at jetties.

At inlet jetties on the Florida east coast there is accretion to the north and erosion to the south. In certain instances sand tending to pass by an inlet is carried from the ocean through the inlet and deposited in the bay to the rear. This latter phenomenon tends to destroy the purpose for which the inlet was constructed. An attempt toward the solution of the beach erosion problem in the vicinity of an inlet has been attempted at the Boynton Inlet, where the sand is pumped from the north side to a system of protective works on the south side. Conditions to the south have been considerably improved since pumping began. No doubt general practice of this kind will provoke objections from property owners on the north side of the inlet. In certain instances it may be feasible to pump sand that has drifted from the ocean into the bay back into the ocean to the south of the inlet.

It should be emphasized that in many instances a beach erosion problem would not exist had inlets not been constructed. Therefore, serious consideration must be given by the civil engineer, during the design of an inlet, to its ultimate effects on adjacent property.

Colonel Brown states that the progressive reduction in length of groins toward the ends of a protective system has proved successful in some instances, whereas in other cases it has not been effective at all. In South Florida attempts have been made toward progressive reduction, but they have not been successful. It should be realized that after a system of protective works has been constructed on the east coast of Florida its extension to the south will be required eventually.

The requirement advocated by Colonel Brown that a groin be sand tight is correct. This type has proved to be the most successful in Florida. Permeable groins have been constructed, but there have been instances in which they have been converted to solid ones.

To adopt a ratio between the length of the groin and spacing of 1 to 3, which is one limit of Colonel Brown's suggested spacing, would be a radical change from the ratio now used in southeast Florida where an economical length of groin is approximately two-thirds of the spacing.

The length of a groin plays a large part in the extent to which a beach will build up. At Miami Beach, where approximately 9 000 ft of ocean front are protected by city-constructed works, groins in one system have a length of 170 ft, in a second a length of 200 ft, and in a third a length of 250 ft. More extensive lengths of beach have been built up in the areas having longer groins. In all these systems the two-thirds rule has been followed. Naturally, in many instances, such considerations as depth of water, wave action, and structural stability will determine the length.

Other factors such as wave height and life of the structure should be considered, in addition to those mentioned by the author, to determine the profile of a groin. The inshore end of a groin should be above normal wave height, not only because it is desirable to have an available beach at high tides, but so that such a beach will be formed that ordinary waves will break on it, and not against the bulkhead. The back-lash of waves breaking against a sea-wall

and meeting the incoming waves will not only retard the building up of a beach, but may prevent the making of one. Where it is desired that the beach be made naturally, it would not be necessary to construct the inshore groin elevation above a point where occasional heavy storms would deposit sand.

Because of the expense involved in protecting beaches it is necessary, as Colonel Brown states, to study the history of existing protective works. In Florida, problems of deterioration and structural stability enter into the economics of groin design. In localities such as Palm Beach, where there is a strong surf, steel-piling has been badly eroded by sand action. At Miami Beach, where there is comparatively light surf, there are only a few signs of steel erosion. When difficulties of this kind are anticipated creosoted timber sheeting, so designed that it can be renewed readily, should partly encase the steel groins. Corrosion of steel in groins appears to be of minor importance. The sand covering at the inshore end, and the continuous wetting and accumulation of barnacles on the outer end, are factors that retard corrosion.

In southern waters, toredo attack creosoted timber through field cuts and drill holes. The life of creosoted timber can be increased by requiring all cutting and drilling to be done before treatment. Special care during construction has proved this method to be feasible. At Miami Beach, where there is a 2.5 ft mean tidal range, steel groins braced by creosoted timber wales and piles have been built at such an elevation, in some instances, that normal water does not touch the timber wales. Groins of this design, with an inshore elevation of 6.5 ft and an offshore elevation of 3.5 ft above mean low water, have functioned successfully.

The forming of deep water at the end of a groin, by eddies, makes special attention necessary for structural stability. Longer sheet-piles and heavier braces are advisable in the affected areas. The placing of granite blocks, if such blocks are available at a reasonable cost, should be effective in strengthening the ends.

Colonel Brown sets forth four important factors that should be given consideration in determining the proper height for a bulkhead. The problem of determining the height of a bulkhead has been comparatively simple at Miami Beach. The top coincides approximately with the backshore terrace. The top elevation is 8.5 ft above mean low water, which is above the ocean water at all times except during severe hurricane periods. To build a bulkhead to hold back hurricane seas would not only be unjustifiable economically, but would impair the appearance of the beach and the view of the ocean.

At Palm Beach the problem is more difficult. In that town there is a highway along the backshore terrace at 16 ft above mean low water. Generally the bulkhead has been constructed close to the highway at approximately the elevation of the highway. In 1937, a bulkhead was constructed to the seaward of the customary location, at an elevation of about 9 ft above mean low water. Naturally, this method of construction is cheaper than the usual type. The height is sufficient to withstand ordinary seas. There is not only an expansive beach in front, built up by the groins, but there is also considerable sand space between the bulkhead and the highway. During hurricane seas there

would possibly be some erosion to the rear of the bulkhead. However, if the slope of the sand between the bulkhead and the highway were constructed at a grade not much steeper than the slope of the beach seaward from the bulkhead the tendency toward erosion could be diminished. By virtue of the economies involved and the additional available beach space, construction of this type is worthy of special attention and study.

Corrosion of a steel piling bulkhead does not appear to present a serious problem as only the exposed portions are affected. Since the exposed portions are of shallow depth in a well-designed system an inexpensive concrete cap can be placed to replace a corroded portion.

Colonel Brown has made an excellent contribution to the science of beach protection and his excellent paper deserves a place in the library of every interested civil engineer.